Dynamic spectral analysis based on an autoregressive model with time-varying coefficients

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Abstract - **A method of dynamic spectral analysis based on an autoregressive model with time-varying coefficients is presented. For parameter estimation a method simpler than the Kalman approach is introduced. The capability of the estimation procedure is demonstrated by applying it to simulated data.**

I. Introduction

In general, physiological signals are non-stationary. Thus conventional spectral analysis techniques cannot be applied. In this papers we suggest using an autoregressive model with time-varying coefficients to investigate the dynamic spectral properties of such signals.

An autoregressive model with time-varying coefficients of order p is defined by

$$Y_t = a_{1,t}Y_{t-1} + a_{2,t}Y_{t-2} + \ldots + a_{p,t}Y_{t-p} + Z_t, \qquad (1.1)$$

where $\{Z_t\}$ is a white noise process with variance σ_t^2. Provided the model parameters vary slowly through time and the roots of the polynomial $1 - a_{1,t}z - \ldots - a_{p,t}z^p$ are outside the unit circle of the complex plane the process is locally stationary at time t. It follows that the power spectrum of $\{Y_t\}$ at time t is well-defined and given by

$$S_{Y,t}(\omega) = \frac{\sigma^2}{2\pi} \frac{1}{\left|1 - a_{1,t}e^{-i\omega} - \ldots - a_{p,t}e^{-i\omega p}\right|^2} \qquad (1.2)$$

for $-\pi \le \omega \le \pi$. Full details are given in [1].

II. Methods

Assuming the model introduced (1.1) the following adaptive estimation equations are presented in [2]:

$$\hat{a}_{j,t} = \begin{cases} 0 & \text{for } t \le j,\ j = 1,\ldots,p \\ \hat{a}_{j,t-1} + c_t e_t Y_{t-j} & \text{for } t > j,\ j = 1,\ldots,p \end{cases}. \qquad (2.1)$$

where

$$e_t = Y_t - \hat{a}_{1,t-1}Y_{t-1} - \ldots - \hat{a}_{p,t-1}Y_{t-p} \qquad (2.2)$$

is the prediction error at time t. The constant c_t is determined by

$$c_t = \frac{f}{\sqrt{\hat{\sigma}^2_{Y,t}}} \qquad (2.3)$$

where $\hat{\sigma}^2_{Y,t}$ is the estimated variance of the process $\{Y_t\}$ at time t given by

$$\hat{\sigma}^2_{Y,t} = (1 - c_s)\hat{\sigma}^2_{Y,t-1} + c_s Y_t^2, \quad (0 < c_s < 1) \qquad (2.4)$$

for properly chosen c_s. The constant f determines the speed of adaptation. Similarly, we estimate the variance of the process $\{Z_t\}$ at time t by

$$\hat{\sigma}^2_{Z,t} = (1 - c_s)\hat{\sigma}^2_{Z,t-1} + c_s e_t^2. \qquad (2.5)$$

Thus we obtain the following estimate of the power spectrum of $\{Y_t\}$ at time t

$$\hat{S}_{Y,t}(\omega) = \frac{\hat{\sigma}^2_{Z,t}}{2\pi} \frac{1}{\left|1 - \hat{a}_{1,t}e^{-i\omega} - \ldots - \hat{a}_{p,t}e^{-i\omega p}\right|^2}. \qquad (2.6)$$

III. Results

In order to evaluate the performance of the estimation procedure data were simulated using a set of stationary autoregressive models of order 2, where the coefficients were changed stepwise (with constant variance of the white noise process). Fig. 1 shows the time course of parameter $a_{1,t}$ for $t = 1,\ldots,2000$.

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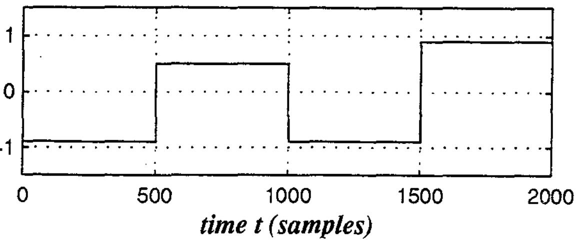

Fig. 1: Model coefficient $a_{1,t}$ *of simulated signal* Y_t

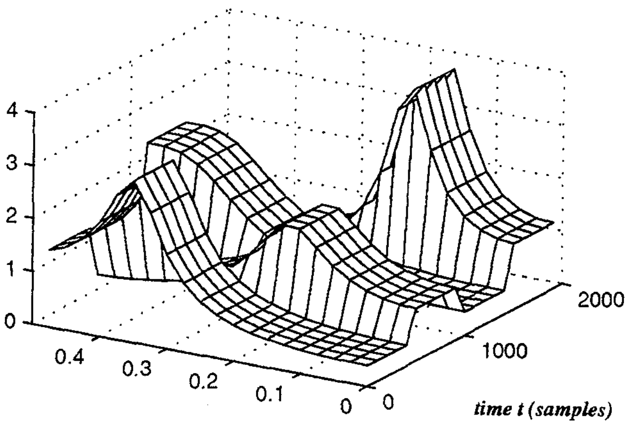

Fig. 2: Power spectra $S_{Y,t}(\omega)$ *of simulated signal* Y_t

The power spectra of the models used for simulation are shown in Fig. 2.

The simulated data were used to estimate the model coefficients, applying the adaptive algorithm presented above. The time course of the estimated model coefficient $\hat{a}_{1,t}$ is presented in figure 3.

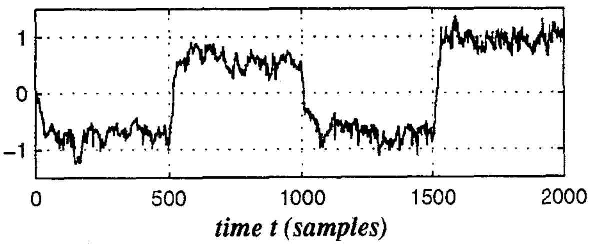

Fig. 3: Estimated model coefficient $\hat{a}_{1,t}$

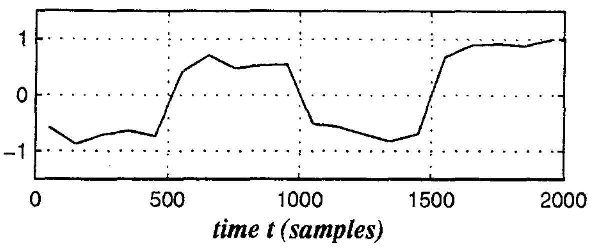

Fig. 4: Average estimated model coefficient $\hat{a}_{1,t}$

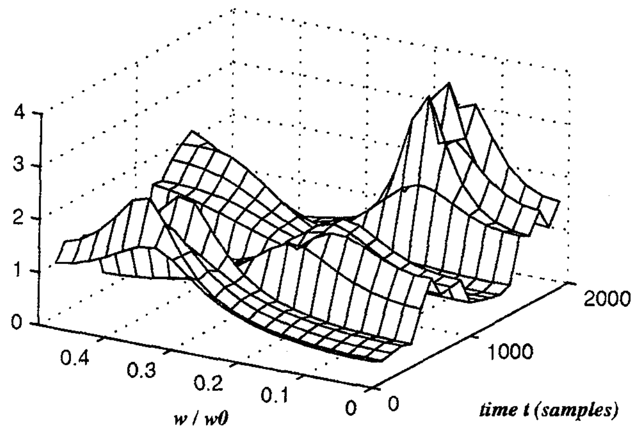

Fig. 5: Estimated power spectra $\hat{S}_{Y,t}(\omega)$

Figure 4 shows the parameter estimates averaged across a set of 100 consecutive samples. These averaged estimates were substituted into (2.6) to obtain the estimated power spectra $\hat{S}_{Y,t}(\omega)$ corresponding to $S_{Y,t}(\omega)$, which are shown in Fig. 5.

IV. Discussion

For choosing a proper value for f in (2.3), a compromise had to be found. If f is small, the adaptation on parameter changes is slow but the variance of the estimates is low. If f is large, adaptation will be fast, however, variance of the estimates will increase too. In the example above f was chosen to be 0.01. As long as the variance $\hat{\sigma}^2_{Y,t}$ varies slowly, the value of c_s in (2.4) and (2.5) is uncritical. Here $c_s = 0.05$ was chosen.

In [3] a Kalman filtering approach to estimating coefficients in autoregressive models is introduced. The computational effort with this approach is much higher compared to the method presented here.

However, in certain applications computation time is critical and/or the improved accuracy is not needed. This is the case, when analyzing EEG on-line, which is intended to be done with this method. (A presentation of this paper would include an application of this method to event-related EEG.)

References

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