Primal-Dual Non-Smooth Friction for Rigid Body Animation: Supplementary Paper

YI-LU CHEN, ISTA, Austria
MICKAËL LY, ISTA, Austria
CHRIS WOJTAN, ISTA, Austria

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1 SCHUR COMPLEMENTS OF THE NEWTON SYSTEM

We first start by examining the min-max problem

$$\begin{aligned} \min_{\mathbf{v}} \max_{\mathbf{r}} \; & \frac{1}{2}(\mathbf{v}-\tilde{\mathbf{v}})^{\mathsf{T}}\mathbf{M}(\mathbf{v}-\tilde{\mathbf{v}}) + \tilde{U}(\mathbf{v}) - \mathbf{v}^{\mathsf{T}}\mathbf{H}^{\mathsf{T}}\mathbf{r} \\ \text{s.t.} \quad & -\mathbf{r}_N \leq 0 \\ & \mathbf{c}(\mathbf{r}_T, \mathbf{s}) \leq 0. \end{aligned} \tag{1}$$

We start by noting that the from Eq. 1, we solve for a *minimization* with regard to $\mathbf{v}$ and a *maximization* with regard to $\mathbf{r}$, and since λ are Lagrange multipliers on $\mathbf{r}$, we solve a *minimization* with regard to λ. A Newton's method with a regularizer ϵ will then bring the search step closer to gradient *descent* for $\mathbf{v}$ and λ, and gradient *ascent* for $\mathbf{r}$. Thus we have the regularized system:

$$\begin{bmatrix} \mathbf{A}+\epsilon\mathbb{I} & -\mathbf{H}^{\mathsf{T}} & \mathbf{0} \\ -\mathbf{H} & -\sum_i \lambda_i \nabla^2 \mathbf{b}_i - \epsilon\mathbb{I} & -\nabla\mathbf{b} \\ \mathbf{0} & \mathbf{\Lambda}\nabla\mathbf{b}^{\mathsf{T}} & \mathbf{B}-\epsilon\mathbf{\Lambda}\mathbb{I} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{v} \\ \Delta\mathbf{r} \\ \Delta\lambda \end{bmatrix} = - \begin{bmatrix} \rho_{\mathbf{v}} \\ \rho_{\mathbf{r}} \\ \rho^{\lambda} \end{bmatrix}. \tag{2}$$

where the signs of ϵ reflect the gradient descent or ascent of the variable. The extra scale $-\mathbf{\Lambda}$ in the lower-right corner follows from the fact that the Hessian of a scalar function must be symmetric, and multiplying the third row of Eq. 2 by $-\mathbf{\Lambda}^{-1}$ results in a symmetric system. Note here that $\mathbf{B}$ and $\epsilon\mathbf{\Lambda}\mathbb{I}$ are diagonal matrices with non-positive and non-negative entries respectively. Following conventions of primal–dual interior point methods [Boyd and Vandenberghe 2004], we limit the step size in each Newton step such that the variables lie strictly within the boundaries of the constraints. $\mathbf{B}-\epsilon\mathbf{\Lambda}\mathbb{I}$ hence trivially invertible, since all diagonal entries are negative. We can then eliminate the third row by substituting

$$\Delta\lambda = -\underbrace{(\mathbf{B}-\epsilon\mathbf{\Lambda}\mathbb{I})^{-1}}_{\mathbf{D}}(\rho_{\lambda} + \mathbf{\Lambda}\nabla\mathbf{b}^{\mathsf{T}}\Delta\mathbf{r}) \tag{3}$$

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into in the second line, yielding the system

$$\begin{bmatrix} \mathbf{A}+\epsilon\mathbb{I} & -\mathbf{H}^\intercal \\ -\mathbf{H} & \underbrace{\begin{matrix} -\sum_i \lambda_i \nabla^2 \mathbf{b}_i - \epsilon\mathbb{I} \\ +\nabla\mathbf{b}\mathbf{D}\Lambda\nabla\mathbf{b}^\intercal \end{matrix}}_{-\mathbf{K}^{-1}} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{v} \\ \Delta\mathbf{r} \end{bmatrix} = -\begin{bmatrix} \rho_{\mathbf{v}} \\ \underbrace{\rho_{\bar{\mathbf{r}}} + \nabla\mathbf{b}\mathbf{D}\rho_\lambda}_{\mathbf{x}} \end{bmatrix}. \tag{4}$$

Observe again that both $\sum_i \lambda_i \nabla^2 \mathbf{b}_i$ and $-\nabla\mathbf{b}\mathbf{D}\Lambda\nabla\mathbf{b}^\intercal$ are symmetric positive-semidefinite matrices, thus with the addition of $\epsilon\mathbb{I}$, this block becomes symmetric positive-definite, and thus also invertible. By defining

$$\mathbf{K} = \left(\sum_i \lambda_i \nabla^2 \mathbf{b}_i + \sum_i \frac{\lambda_i}{-\mathbf{b}_i + \lambda_i \epsilon} (\nabla\mathbf{b}_i)(\nabla\mathbf{b}_i)^\intercal + \epsilon\mathbb{I} \right)^{-1}$$

we can further eliminate the second row in the same manner:

$$\Delta\mathbf{r} = \mathbf{K}(\mathbf{x} - \mathbf{H}\Delta\mathbf{v}) \tag{5}$$

followed by

$$(\mathbf{A} + \epsilon\mathbb{I} + \mathbf{H}^\intercal\mathbf{K}\mathbf{H})\Delta\mathbf{v} = -\rho_{\mathbf{v}} + \mathbf{H}^\intercal\mathbf{K}\mathbf{x}. \tag{6}$$

Finally, assuming that $\mathbf{A}$ is symmetric positive definite, the left hand side is again invertible, and we can now solve for $\Delta\mathbf{v}$, $\Delta\mathbf{r}$, and $\Delta\lambda$.

2 ALART–CURNIER FUNCTION

Alart and Curnier [1991] proposed the following function

$$\begin{array}{rccc} \Phi^{\mathrm{AC}}: & \mathbb{R}^3 \times \mathbb{R}^3 & \rightarrow & \mathbb{R}^3 \\ & (\mathbf{u}, \mathbf{r}) & \mapsto & \begin{bmatrix} \mathrm{proj}_{\mathbb{R}_{\geq 0}}(\mathbf{r}_N - \rho_N \mathbf{u}_N) - \mathbf{r}_N \\ \mathrm{proj}_{\mathcal{B}(\mu\mathbf{r}_N)}(\mathbf{r}_T - \rho_T \mathbf{u}_T) - \mathbf{r}_T \end{bmatrix} \end{array} \tag{7}$$

with proj_C denoting the projection on the set C, $\mathcal{B}(\mu\mathbf{r}_N)$ the ball centered in 0 of radius $(\mu\mathbf{r}_N)$ and ρ_N, ρ_T two positive constants, usually taken equal to 1.

Given at a contact point the local relative velocity $\mathbf{u} \in \mathbb{R}^3$ and the frictional contact force $\mathbf{r} \in \mathbb{R}^3$, $\Phi^{\mathrm{AC}}(\mathbf{u}, \mathbf{r}) \geq 0$ and most importantly

$$\Phi^{\mathrm{AC}}(\mathbf{u}, \mathbf{r}) = 0 \Leftrightarrow (\mathbf{u}, \mathbf{r}) \text{ satisfy the Coulomb contact law.} \tag{8}$$

To extend the Alart–Curnier function to Baumgarte Stabilization:

$$-(\mathbf{H}\mathbf{v})_N - \gamma\mathbf{r}_N - \sum_i \lambda_i \nabla_{\mathbf{r}_N} \mathbf{b}_i(\mathbf{r}) = \frac{e}{\delta t}\varphi. \tag{9}$$

, we extend the normal part of the equation to

$$\mathrm{proj}_{\mathbb{R}_{\geq 0}} \left(\mathbf{r}_N - \rho_N(\mathbf{u}_N + \gamma\mathbf{r} + \frac{e}{\delta t}\varphi) \right) - \mathbf{r}_N \tag{10}$$

One can then see that for this function to equal 0, either $\mathbf{u}_N + \gamma\mathbf{r} + \frac{e}{\delta t}\varphi = 0$ and $\mathbf{r}_N \geq 0$, or $\mathbf{r}_N = 0$ and $\mathbf{u}_N + \gamma\mathbf{r} + \frac{e}{\delta t}\varphi \geq 0$, satisfying Eq. 9. Conversely, if Eq. 9 is satisfied with $0 \leq \lambda_i \perp \mathbf{u}_N + \gamma\mathbf{r} + \frac{e}{\delta t}\varphi \geq 0$, then Eq. 10 is satisfied.

3 CONVERGENCE STUDIES

3.1 Methodology

Rigid–IPC. The friction model used in Rigid–IPC [Ferguson et al. 2021] is described in [Li et al. 2020] and we refer below to the corresponding sections or equations of that paper for a more detailed description.

For this code, we considered the 3 following parameters:

- ε_v is the characteristic speed controlling the smoothing of the friction law. The smaller this value is, the more the friction model tends towards the non-smooth model (See their Eq. 13);
- Then, ε_d is the threshold to which the Incremental Potential minimisation (including the momentum, the log contact barriers, the smoothed lagged friction) is solved. The smaller the value, the more accurate the integration (See their Sec. 4.3);
- Finally, ε_μ controls the lagged iterations convergence. The smaller the value, the better the agreement between the barrier contact force and its lagged value (See their Eq. 17).

Our method. For our method, we consider the following parameters:

- ϵ_τ, the exit criterion for the Newton iterations.
- ϵ_p, ϵ_s, the friction constraint perturbation parameters. Typically, a larger perturbation ϵ_p will require a ϵ_s to be stable. Here we choose $\epsilon_p = \epsilon_s$ for all examples.

3.2 Cube on an inclined plane

A cube of mass m is placed slightly hovering and parallel to a plane inclined by $\theta = \tan^{-1}(0.5) \approx 26.565°$, such that the friction forces will prevent sliding due to gravity g only if the friction coefficient $\mu \leq 0.5$. Otherwise, for $\mu < 0.5$, the contact forces are

$$\mathbf{r}_N = mg\cos\theta \tag{11a}$$

$$\mathbf{r}_T = -\mu\mathbf{r}_N \tag{11b}$$

leading to an acceleration in the tangent plane equal to

$$a_\mathrm{T} = g(\sin\theta - \mu\cos\theta). \tag{12}$$

We perform this simulation for different μ and vary the three parameters described in Sec. 3.1. The detailed values are in the tables Tab. 1 to Tab. 6 for Rigid–IPC and Tab. for our method, and we show the corresponding trajectories with $\mu = 0.45$ in Fig. 1 and Fig. 2 respectively. For that example, we observe for both methods a good convergence as more accuracy is requested.

Regarding Rigid–IPC ε_d and ε_μ allow the trajectory to converge, as reflected through the convergence of the mean velocity and acceleration and ε_v allows to generate more accurate friction forces for low velocities, allowing to get the correct acceleration (see in particular Tab. 3 for $\mu = 0.499$) and a smaller error on the sliding velocity for $\mu > 0$. For our method, the perturbed and shifted cone overestimates the friction, as can be seen in Tab. 7. Lowering the perturbation converges to the analytical solution as expected.

3.3 House of cards

In that example, we consider a 5-story tall house of cards, with the cards initially barely not in contact, and we are interested in studying the qualitative behavior of the house of cards as we vary the friction coefficient and the other

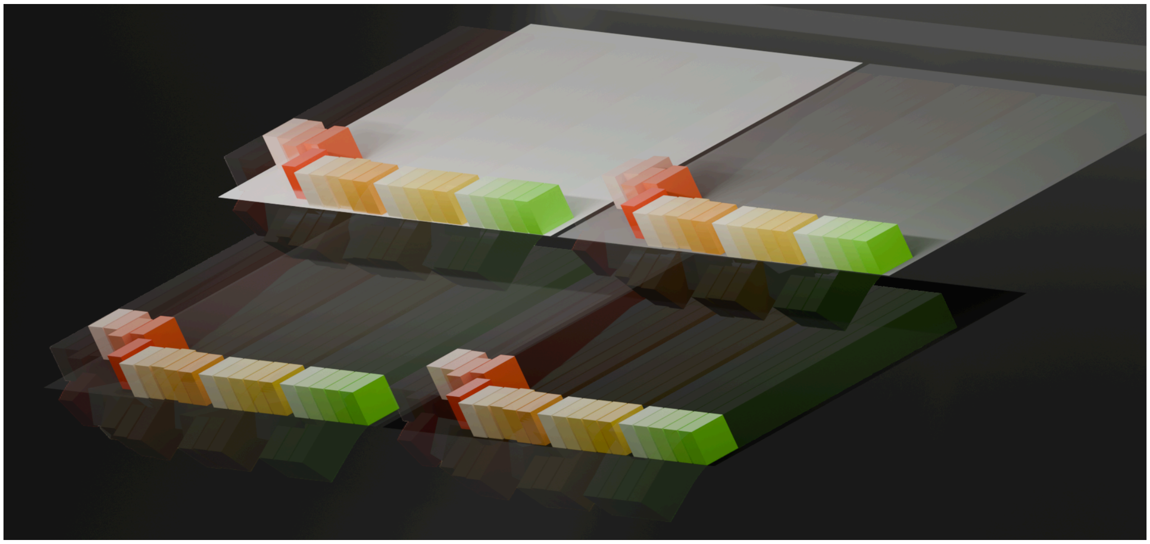

Fig. 1. Trajectories of cubes on inclined planes simulated with Rigid–IPC with $\mu = 0.45$ with an emphasis on the "finish line". A darker ground denotes a stiffer friction model (smaller ε_v). The saturation (from white to colored) indicates a more accurate Newton solve (lower ε_d tolerance) and the hue (from red to green) a more accurate friction solve (lower ε_μ tolerance). The corresponding detailed values are in the Tab. 1. Overall, requiring a more accurate simulation allows the convergence towards the right acceleration and toppling behavior.

aforementioned parameters. We report in Tab. 8 to Tab. 12 for Rigid–IPC and in Tab. 18 to Tab. 19 for our method the behaviors, classified in four modes:

- *Total Collapse*(TC): the whole house of cards collapsed initially;
- *Cascade Collapse*(CC): the top two cards toppled in a way that made some other part of the house of cards to collapse;
- *Partial Collapse*(PC): only the top two cards fell down;
- *Standing*(S): no cards fell down.

For Rigid–IPC, we observed that a certain level of accuracy is required to observe qualitative differences across different μ, especially for low friction where the house of cards may look overly stable for not accurate enough solves. However, this led to non-converging timesteps as cards start to generate a lot more contacts (Fig. 3a). As such, we report in the Tables the last converged timestep on a $3s$ simulation. We also tried to decrease the timestep from $10ms$ to $5ms$, which help with the convergence but did not solve the problem entirely, especially for stiff ε_v (Tab. 13 to Tab. 17, Fig. 3b).

For our method, we observe that a high Newton tolerance ϵ_τ can cause the house of cards to unexpectedly collapse, though this is remedies simply with a lower tolerance. In most cases, our perturbation parameters do not affect the overall behavior of the house of cards, though the collapse mode can switch between total collapse and cascade collapse when $\mu = 0.4$ Tab. 18c. This inconsistency is not present after lowering δt to $5ms$, where the collapsing behaviour becomes consistent across perturbation parameters once ϵ_τ is lower than 10^{-4}.

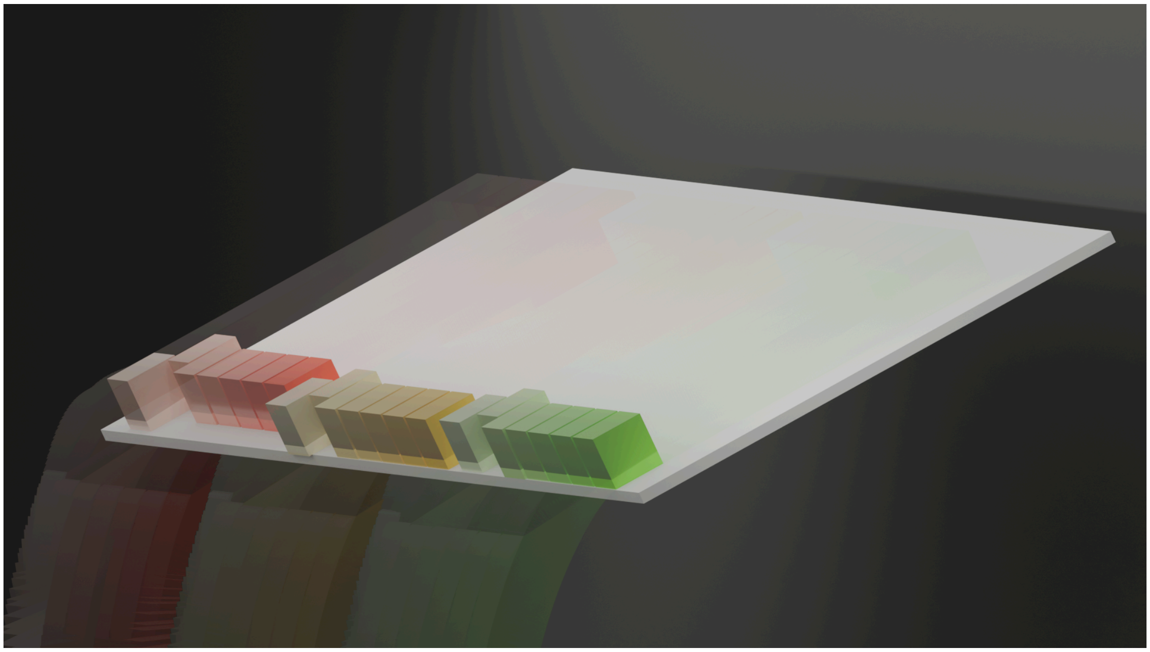

Fig. 2. Trajectories of cubes on inclined planes simulated with our method with $\mu = 0.45$ with an emphasis on the "finish line". The saturation (from white to colored) indicates a more accurate Newton solve (lower ϵ_τ tolerance) and the hue (from red to green) indicate lower perturbation parameters(lower ϵ_p and ϵ_s). The corresponding detailed values are in the Tab. 7. An overly large ϵ_p and ϵ_s can result in overestimation of friction forces, but lowering ϵ_τ, ϵ_p, and ϵ_s converges to the analytical solution.

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$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(0.000556, -4.45 \cdot 10^{-13})$	$(0.872, 0.329)$	$(1.14, 0.433)$	$(1.15, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(7.91, 500)$	$(6.96, 500)$	$(13.6, 500)$	$(15, 500)$
0.001	$(0.000556, -4.45 \cdot 10^{-13})$	$(0.878, 0.325)$	$(1.14, 0.432)$	$(1.16, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(4.74, 500)$	$(4.63, 500)$	$(6.45, 500)$	$(21.7, 500)$
0.0001	$(0.000556, -4.45 \cdot 10^{-13})$	$(0.936, 0.356)$	$(1.13, 0.429)$	$(1.16, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(2.5, 11)$	$(2.84, 11)$	$(3.53, 12)$	$(60.3, 500)$
$1 \cdot 10^{-5}$	$(0.112, 0.0278)$	$(0.884, 0.336)$	$(1.15, 0.435)$	$(1.15, 0.437)$	$(1.16, 0.438)$
	$(1.08, 2)$	$(1.89, 4)$	$(2.41, 6)$	$(2.57, 9)$	$(4.18, 500)$
$1 \cdot 10^{-6}$	$(1.12, 0.41)$	$(1.1, 0.41)$	$(1.15, 0.436)$	$(1.15, 0.437)$	$(1.16, 0.438)$
	$(1, 2)$	$(1.03, 2)$	$(1.73, 4)$	$(2.07, 6)$	$(2.47, 7)$

(a) $\varepsilon_v = 1 \cdot 10^{-3}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5.56 \cdot 10^{-5}, -4.51 \cdot 10^{-16})$	$(0.873, 0.329)$	$(1.14, 0.433)$	$(1.15, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(8.05, 500)$	$(7.3, 500)$	$(9.02, 500)$	$(23.5, 500)$
0.001	$(5.56 \cdot 10^{-5}, -4.18 \cdot 10^{-16})$	$(0.873, 0.325)$	$(1.14, 0.431)$	$(1.16, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(5.06, 407)$	$(6.39, 500)$	$(6.37, 500)$	$(46.4, 500)$
0.0001	$(5.56 \cdot 10^{-5}, -3.86 \cdot 10^{-16})$	$(0.932, 0.359)$	$(1.13, 0.429)$	$(1.16, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(2.53, 10)$	$(2.82, 13)$	$(3.54, 12)$	$(61.1, 500)$
$1 \cdot 10^{-5}$	$(5.56 \cdot 10^{-5}, -5.03 \cdot 10^{-16})$	$(0.884, 0.336)$	$(1.15, 0.435)$	$(1.15, 0.437)$	$(1.16, 0.438)$
	$(1, 2)$	$(1.92, 4)$	$(2.52, 6)$	$(2.58, 9)$	$(4.25, 500)$
$1 \cdot 10^{-6}$	$(1.12, 0.41)$	$(1.1, 0.41)$	$(1.15, 0.436)$	$(1.15, 0.437)$	$(1.16, 0.438)$
	$(1, 2)$	$(1.03, 2)$	$(1.72, 4)$	$(2.07, 6)$	$(2.47, 7)$

(b) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5.56 \cdot 10^{-6}, -3.51 \cdot 10^{-17})$	$(0.87, 0.329)$	$(1.14, 0.432)$	$(1.15, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(11.6, 500)$	$(8.22, 500)$	$(22.9, 500)$	$(15.7, 500)$
0.001	$(5.56 \cdot 10^{-6}, -1.44 \cdot 10^{-16})$	$(0.879, 0.326)$	$(1.14, 0.431)$	$(1.15, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(5.58, 500)$	$(6.83, 500)$	$(7.18, 18)$	$(9.67, 500)$
0.0001	$(5.56 \cdot 10^{-6}, 1.61 \cdot 10^{-17})$	$(0.933, 0.361)$	$(1.13, 0.429)$	$(1.16, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(2.5, 9)$	$(2.83, 12)$	$(3.58, 15)$	$(66.1, 500)$
$1 \cdot 10^{-5}$	$(5.56 \cdot 10^{-6}, 5.56 \cdot 10^{-17})$	$(0.884, 0.336)$	$(1.15, 0.435)$	$(1.15, 0.437)$	$(1.16, 0.438)$
	$(1.01, 3)$	$(1.88, 4)$	$(2.42, 6)$	$(2.6, 9)$	$(4.19, 500)$
$1 \cdot 10^{-6}$	$(1.12, 0.409)$	$(1.09, 0.409)$	$(1.15, 0.436)$	$(1.15, 0.437)$	$(1.16, 0.438)$
	$(1, 2)$	$(1.03, 2)$	$(1.7, 4)$	$(2.05, 6)$	$(2.47, 7)$

(c) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5.56 \cdot 10^{-7}, -2.53 \cdot 10^{-18})$	$(0.875, 0.335)$	$(1.14, 0.433)$	$(1.15, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(16, 500)$	$(17.5, 500)$	$(13.3, 500)$	$(14.7, 500)$
0.001	$(5.56 \cdot 10^{-7}, 3.27 \cdot 10^{-17})$	$(0.879, 0.326)$	$(1.14, 0.431)$	$(1.16, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(7.71, 500)$	$(6.48, 13)$	$(6.64, 500)$	$(9.41, 459)$
0.0001	$(5.56 \cdot 10^{-7}, -1.5 \cdot 10^{-17})$	$(0.93, 0.357)$	$(1.13, 0.429)$	$(1.16, 0.438)$	$(1.16, 0.438)$
	$(1, 2)$	$(2.51, 11)$	$(2.85, 12)$	$(3.55, 12)$	$(62, 500)$
$1 \cdot 10^{-5}$	$(5.56 \cdot 10^{-7}, -1.84 \cdot 10^{-17})$	$(0.878, 0.339)$	$(1.15, 0.435)$	$(1.15, 0.437)$	$(1.16, 0.438)$
	$(1, 2)$	$(1.88, 6)$	$(2.43, 6)$	$(2.75, 9)$	$(3.14, 10)$
$1 \cdot 10^{-6}$	$(1.12, 0.409)$	$(1.1, 0.41)$	$(1.15, 0.436)$	$(1.15, 0.437)$	$(1.16, 0.438)$
	$(1, 2)$	$(1.03, 2)$	$(1.72, 4)$	$(2.07, 6)$	$(2.48, 7)$

(d) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 1. Cube on a slope, table for $\mu = 0.45$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the average velocity and acceleration ($\approx 0.438 m \cdot s^{-1}$ in theory), the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(0.000505, -3.35 \cdot 10^{-13})$	$(0.000904, -1.3 \cdot 10^{-12})$	$(0.102, 0.0384)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3.02, 8)$	$(8.09, 46)$	$(10.1, 500)$	$(9.23, 41)$
0.001	$(0.000505, -3.35 \cdot 10^{-13})$	$(0.000904, -1.29 \cdot 10^{-12})$	$(0.102, 0.0376)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1.01, 3)$	$(3, 4)$	$(6.64, 20)$	$(7.19, 22)$	$(7.74, 20)$
0.0001	$(0.000505, -3.34 \cdot 10^{-13})$	$(0.000904, -1.3 \cdot 10^{-12})$	$(0.102, 0.0379)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1.01, 3)$	$(3, 4)$	$(5.07, 15)$	$(5.91, 36)$	$(6.4, 13)$
$1 \cdot 10^{-5}$	$(0.000763, -7.93 \cdot 10^{-13})$	$(0.000904, -1.28 \cdot 10^{-12})$	$(0.102, 0.0384)$	$(0.114, 0.043)$	$(0.115, 0.0438)$
	$(1, 2)$	$(2.01, 4)$	$(3.35, 5)$	$(4.04, 5)$	$(5.08, 6)$
$1 \cdot 10^{-6}$	$(0.0781, 0.00765)$	$(0.0704, 0.014)$	$(0.105, 0.0394)$	$(0.114, 0.0432)$	$(0.115, 0.0438)$
	$(1, 2)$	$(1, 2)$	$(1.53, 4)$	$(2.68, 4)$	$(3.15, 6)$

(a) $\varepsilon_v = 1 \cdot 10^{-3}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5.05 \cdot 10^{-5}, -3.41 \cdot 10^{-16})$	$(9.03 \cdot 10^{-5}, -1.3 \cdot 10^{-15})$	$(0.102, 0.038)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3.02, 8)$	$(7.96, 45)$	$(9.86, 500)$	$(9.22, 56)$
0.001	$(5.05 \cdot 10^{-5}, -3.73 \cdot 10^{-16})$	$(9.03 \cdot 10^{-5}, -1.05 \cdot 10^{-15})$	$(0.101, 0.0372)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1.01, 3)$	$(3, 4)$	$(6.61, 23)$	$(8.03, 500)$	$(7.79, 21)$
0.0001	$(5.05 \cdot 10^{-5}, -3.25 \cdot 10^{-16})$	$(9.03 \cdot 10^{-5}, -1.16 \cdot 10^{-15})$	$(0.101, 0.0371)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1.01, 3)$	$(3, 4)$	$(5.14, 14)$	$(6.02, 14)$	$(6.59, 13)$
$1 \cdot 10^{-5}$	$(5.05 \cdot 10^{-5}, -2.41 \cdot 10^{-16})$	$(9.03 \cdot 10^{-5}, -1.47 \cdot 10^{-15})$	$(0.102, 0.0381)$	$(0.114, 0.043)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3, 4)$	$(3.46, 5)$	$(4.14, 5)$	$(5.2, 6)$
$1 \cdot 10^{-6}$	$(0.0753, 0.00709)$	$(9.03 \cdot 10^{-5}, -1.09 \cdot 10^{-15})$	$(0.105, 0.0389)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1, 2)$	$(2, 3)$	$(1.53, 4)$	$(2.69, 5)$	$(3.15, 6)$

(b) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5.05 \cdot 10^{-6}, -4.49 \cdot 10^{-18})$	$(9.03 \cdot 10^{-6}, -3.17 \cdot 10^{-18})$	$(0.101, 0.0378)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3.02, 8)$	$(8.11, 66)$	$(9.34, 500)$	$(9.11, 66)$
0.001	$(5.05 \cdot 10^{-6}, 2.75 \cdot 10^{-17})$	$(9.03 \cdot 10^{-6}, -1 \cdot 10^{-17})$	$(0.101, 0.037)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3, 4)$	$(6.63, 23)$	$(7.29, 43)$	$(7.86, 29)$
0.0001	$(5.05 \cdot 10^{-6}, -1.48 \cdot 10^{-17})$	$(9.03 \cdot 10^{-6}, -4.59 \cdot 10^{-17})$	$(0.101, 0.037)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1.01, 3)$	$(3, 4)$	$(5.3, 57)$	$(6, 14)$	$(6.53, 15)$
$1 \cdot 10^{-5}$	$(5.05 \cdot 10^{-6}, 3.87 \cdot 10^{-17})$	$(9.03 \cdot 10^{-6}, -3.85 \cdot 10^{-17})$	$(0.102, 0.0383)$	$(0.114, 0.043)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3, 4)$	$(3.54, 5)$	$(4.2, 6)$	$(5.26, 6)$
$1 \cdot 10^{-6}$	$(5.05 \cdot 10^{-6}, 2.39 \cdot 10^{-18})$	$(9.03 \cdot 10^{-6}, 8.98 \cdot 10^{-17})$	$(0.102, 0.0387)$	$(0.114, 0.0434)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3, 3)$	$(1.64, 4)$	$(2.8, 5)$	$(3.23, 6)$

(c) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5.05 \cdot 10^{-7}, 3.45 \cdot 10^{-17})$	$(9.03 \cdot 10^{-7}, -2.36 \cdot 10^{-17})$	$(0.101, 0.0377)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3.02, 8)$	$(8.2, 48)$	$(9.29, 500)$	$(9.22, 38)$
0.001	$(5.05 \cdot 10^{-7}, -6.38 \cdot 10^{-17})$	$(9.03 \cdot 10^{-7}, -6.05 \cdot 10^{-17})$	$(0.101, 0.0368)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3.01, 5)$	$(6.67, 26)$	$(7.14, 21)$	$(7.79, 22)$
0.0001	$(5.05 \cdot 10^{-7}, -3.95 \cdot 10^{-17})$	$(9.03 \cdot 10^{-7}, 2.82 \cdot 10^{-17})$	$(0.101, 0.0368)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1.01, 3)$	$(3.01, 5)$	$(5.2, 13)$	$(6.03, 13)$	$(6.53, 14)$
$1 \cdot 10^{-5}$	$(5.05 \cdot 10^{-7}, 8.84 \cdot 10^{-17})$	$(9.03 \cdot 10^{-7}, 7.25 \cdot 10^{-17})$	$(0.102, 0.0383)$	$(0.114, 0.043)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3, 4)$	$(3.56, 5)$	$(4.21, 6)$	$(5.29, 6)$
$1 \cdot 10^{-6}$	$(5.05 \cdot 10^{-7}, 8.62 \cdot 10^{-17})$	$(9.03 \cdot 10^{-7}, 4.57 \cdot 10^{-17})$	$(0.103, 0.0388)$	$(0.114, 0.0433)$	$(0.115, 0.0438)$
	$(1, 2)$	$(3, 3)$	$(1.67, 4)$	$(2.82, 5)$	$(3.26, 6)$

(d) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 2. Cube on a slope, table for $\mu = 0.495$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the average velocity and acceleration ($\approx 0.0438 m \cdot s^{-1}$ in theory), the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(0.000501, -3.27 \cdot 10^{-13})$	$(0.000881, -1.08 \cdot 10^{-12})$	$(0.000997, -2.77 \cdot 10^{-12})$	$(0.0227, 0.00848)$	$(0.0234, 0.0087)$
	$(1, 2)$	$(3.02, 8)$	$(5.01, 9)$	$(5.9, 11)$	$(6.58, 12)$
0.001	$(0.000501, -3.27 \cdot 10^{-13})$	$(0.000881, -1.08 \cdot 10^{-12})$	$(0.000997, -2.78 \cdot 10^{-12})$	$(0.0228, 0.00855)$	$(0.0234, 0.00869)$
	$(1, 2)$	$(3, 4)$	$(5, 6)$	$(4.96, 7)$	$(5.7, 9)$
0.0001	$(0.000501, -3.27 \cdot 10^{-13})$	$(0.000881, -1.08 \cdot 10^{-12})$	$(0.000997, -2.77 \cdot 10^{-12})$	$(0.0228, 0.00855)$	$(0.0234, 0.0087)$
	$(1, 2)$	$(3, 4)$	$(5, 6)$	$(4.46, 7)$	$(5.29, 9)$
$1 \cdot 10^{-5}$	$(0.000753, -7.51 \cdot 10^{-13})$	$(0.000881, -1.06 \cdot 10^{-12})$	$(0.000997, -2.74 \cdot 10^{-12})$	$(0.0226, 0.00848)$	$(0.0234, 0.0087)$
	$(1, 2)$	$(2.01, 4)$	$(4, 6)$	$(3.35, 7)$	$(4.09, 9)$
$1 \cdot 10^{-6}$	$(0.0431, 0.00755)$	$(0.0143, 0.00367)$	$(0.0143, 0.00367)$	$(0.0227, 0.00856)$	$(0.0234, 0.00872)$
	$(1, 2)$	$(1, 2)$	$(1.01, 4)$	$(2.29, 5)$	$(2.98, 6)$

(a) $\varepsilon_v = 1 \cdot 10^{-3}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5.01 \cdot 10^{-5}, -3.75 \cdot 10^{-16})$	$(8.8 \cdot 10^{-5}, -1.32 \cdot 10^{-15})$	$(9.94 \cdot 10^{-5}, -3.23 \cdot 10^{-15})$	$(0.0223, 0.00848)$	$(0.023, 0.0087)$
	$(1, 2)$	$(3.02, 8)$	$(5.02, 13)$	$(6.03, 10)$	$(6.64, 10)$
0.001	$(5.01 \cdot 10^{-5}, -3.78 \cdot 10^{-16})$	$(8.8 \cdot 10^{-5}, -9.87 \cdot 10^{-16})$	$(9.94 \cdot 10^{-5}, -3.77 \cdot 10^{-16})$	$(0.0224, 0.00855)$	$(0.023, 0.00869)$
	$(1, 2)$	$(3, 4)$	$(5, 7)$	$(5.09, 7)$	$(5.79, 7)$
0.0001	$(5.01 \cdot 10^{-5}, -3.57 \cdot 10^{-16})$	$(8.8 \cdot 10^{-5}, -1.03 \cdot 10^{-15})$	$(9.94 \cdot 10^{-5}, -3.61 \cdot 10^{-15})$	$(0.0224, 0.00855)$	$(0.023, 0.00869)$
	$(1.01, 3)$	$(3, 4)$	$(5, 7)$	$(4.92, 7)$	$(5.63, 7)$
$1 \cdot 10^{-5}$	$(5.01 \cdot 10^{-5}, -2.46 \cdot 10^{-16})$	$(8.8 \cdot 10^{-5}, -1.14 \cdot 10^{-15})$	$(9.94 \cdot 10^{-5}, -3.79 \cdot 10^{-15})$	$(0.0224, 0.00855)$	$(0.023, 0.00869)$
	$(1, 2)$	$(3, 4)$	$(5, 6)$	$(3.98, 6)$	$(4.76, 7)$
$1 \cdot 10^{-6}$	$(0.0417, 0.00755)$	$(8.8 \cdot 10^{-5}, -9.72 \cdot 10^{-16})$	$(0.0157, 0.00549)$	$(0.0223, 0.00857)$	$(0.023, 0.00872)$
	$(1, 2)$	$(2, 3)$	$(1.04, 5)$	$(2.33, 5)$	$(2.94, 6)$

(b) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5.01 \cdot 10^{-6}, -2.87 \cdot 10^{-17})$	$(8.8 \cdot 10^{-6}, 6.95 \cdot 10^{-17})$	$(9.94 \cdot 10^{-6}, -2.69 \cdot 10^{-17})$	$(0.0223, 0.00848)$	$(0.023, 0.0087)$
	$(1.01, 3)$	$(3.02, 8)$	$(5.02, 12)$	$(6.04, 10)$	$(6.64, 10)$
0.001	$(5.01 \cdot 10^{-6}, 5.24 \cdot 10^{-18})$	$(8.8 \cdot 10^{-6}, -3.52 \cdot 10^{-17})$	$(9.94 \cdot 10^{-6}, 1.07 \cdot 10^{-16})$	$(0.0224, 0.00854)$	$(0.023, 0.00869)$
	$(1, 2)$	$(3, 5)$	$(5, 6)$	$(5.11, 7)$	$(5.8, 7)$
0.0001	$(5.01 \cdot 10^{-6}, 8.29 \cdot 10^{-17})$	$(8.8 \cdot 10^{-6}, 7.38 \cdot 10^{-17})$	$(9.94 \cdot 10^{-6}, -2.92 \cdot 10^{-17})$	$(0.0224, 0.00854)$	$(0.023, 0.00869)$
	$(1, 2)$	$(3, 5)$	$(5, 6)$	$(4.99, 7)$	$(5.69, 7)$
$1 \cdot 10^{-5}$	$(5.01 \cdot 10^{-6}, 1 \cdot 10^{-16})$	$(8.8 \cdot 10^{-6}, -1.67 \cdot 10^{-17})$	$(0.0139, 0.00242)$	$(0.0224, 0.00855)$	$(0.023, 0.00869)$
	$(1, 2)$	$(3, 4)$	$(3.85, 7)$	$(4.28, 7)$	$(5.05, 7)$
$1 \cdot 10^{-6}$	$(5.01 \cdot 10^{-6}, 1.2 \cdot 10^{-16})$	$(8.8 \cdot 10^{-6}, 1.03 \cdot 10^{-16})$	$(9.94 \cdot 10^{-6}, -2.39 \cdot 10^{-16})$	$(0.0224, 0.00856)$	$(0.023, 0.0087)$
	$(1, 2)$	$(3, 3)$	$(5, 5)$	$(2.91, 5)$	$(3.46, 6)$

(c) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5.01 \cdot 10^{-7}, 6.65 \cdot 10^{-17})$	$(8.8 \cdot 10^{-7}, 1.04 \cdot 10^{-17})$	$(9.94 \cdot 10^{-7}, 4.86 \cdot 10^{-18})$	$(0.0223, 0.00848)$	$(0.023, 0.0087)$
	$(1, 2)$	$(3.02, 8)$	$(5.01, 10)$	$(6.05, 10)$	$(6.65, 10)$
0.001	$(5.01 \cdot 10^{-7}, -1.46 \cdot 10^{-17})$	$(8.8 \cdot 10^{-7}, 4.6 \cdot 10^{-17})$	$(9.94 \cdot 10^{-7}, -2.08 \cdot 10^{-18})$	$(0.0224, 0.00854)$	$(0.023, 0.00869)$
	$(1.01, 3)$	$(3, 5)$	$(5, 6)$	$(5.11, 7)$	$(5.81, 8)$
0.0001	$(5.01 \cdot 10^{-7}, 9.78 \cdot 10^{-18})$	$(8.8 \cdot 10^{-7}, -4 \cdot 10^{-17})$	$(9.94 \cdot 10^{-7}, -1.04 \cdot 10^{-16})$	$(0.0224, 0.00855)$	$(0.023, 0.00869)$
	$(1.01, 3)$	$(3, 4)$	$(5, 6)$	$(4.99, 7)$	$(5.7, 7)$
$1 \cdot 10^{-5}$	$(5.01 \cdot 10^{-7}, 1.83 \cdot 10^{-16})$	$(8.8 \cdot 10^{-7}, -2.27 \cdot 10^{-17})$	$(0.0139, 0.00241)$	$(0.0224, 0.00854)$	$(0.023, 0.00869)$
	$(1, 2)$	$(3, 4)$	$(3.84, 6)$	$(4.36, 6)$	$(5.14, 6)$
$1 \cdot 10^{-6}$	$(5.01 \cdot 10^{-7}, 6.08 \cdot 10^{-17})$	$(8.8 \cdot 10^{-7}, -1.89 \cdot 10^{-17})$	$(0.0152, 0.00305)$	$(0.0224, 0.00854)$	$(0.023, 0.00868)$
	$(1, 2)$	$(3, 3)$	$(2.33, 5)$	$(3.04, 5)$	$(3.53, 6)$

(d) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 3. Cube on a slope, table for $\mu = 0.499$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the average velocity and acceleration ($\approx 0.00877 m \cdot s^{-1}$ in theory), the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(0.0005, -3.25 \cdot 10^{-13})$	$(0.000876, -1.03 \cdot 10^{-12})$	$(0.000939, -1.2 \cdot 10^{-12})$	$(0.000986, -1.46 \cdot 10^{-12})$	$(0.000997, -1.04 \cdot 10^{-9})$
	$(1, 2)$	$(3.02, 8)$	$(4.02, 10)$	$(6.01, 11)$	$(8.01, 11)$
0.001	$(0.0005, -3.25 \cdot 10^{-13})$	$(0.000876, -1.03 \cdot 10^{-12})$	$(0.000939, -1.2 \cdot 10^{-12})$	$(0.000986, -1.45 \cdot 10^{-12})$	$(0.000997, -1.04 \cdot 10^{-9})$
	$(1, 2)$	$(3, 4)$	$(4, 5)$	$(6.01, 10)$	$(8, 8)$
0.0001	$(0.0005, -3.25 \cdot 10^{-13})$	$(0.000876, -1.03 \cdot 10^{-12})$	$(0.000939, -1.2 \cdot 10^{-12})$	$(0.000986, -1.45 \cdot 10^{-12})$	$(0.000997, -1.04 \cdot 10^{-9})$
	$(1.01, 3)$	$(3, 4)$	$(4, 5)$	$(6, 8)$	$(8, 8)$
$1 \cdot 10^{-5}$	$(0.000751, -7.42 \cdot 10^{-13})$	$(0.000876, -1.02 \cdot 10^{-12})$	$(0.000939, -1.19 \cdot 10^{-12})$	$(0.000986, -1.45 \cdot 10^{-12})$	$(0.000997, -1.02 \cdot 10^{-9})$
	$(1, 2)$	$(2, 3)$	$(3.01, 5)$	$(5, 6)$	$(7, 7)$
$1 \cdot 10^{-6}$	$(0.0216, -4.16 \cdot 10^{-5})$	$(0.000973, -3.45 \cdot 10^{-6})$	$(0.000973, -3.1 \cdot 10^{-6})$	$(0.000986, -1.79 \cdot 10^{-12})$	$(0.000997, -1.69 \cdot 10^{-9})$
	$(1, 2)$	$(1, 2)$	$(1.01, 4)$	$(2.01, 4)$	$(4, 5)$

(a) $\varepsilon_v = 1 \cdot 10^{-3}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5 \cdot 10^{-5}, -2.69 \cdot 10^{-16})$	$(8.75 \cdot 10^{-5}, -1.07 \cdot 10^{-15})$	$(9.38 \cdot 10^{-5}, -7.36 \cdot 10^{-16})$	$(9.84 \cdot 10^{-5}, -2.22 \cdot 10^{-15})$	$(0.000112, -8.76 \cdot 10^{-6})$
	$(1, 2)$	$(3.02, 8)$	$(4.02, 10)$	$(6.02, 13)$	$(8.08, 15)$
0.001	$(5 \cdot 10^{-5}, -2.34 \cdot 10^{-16})$	$(8.75 \cdot 10^{-5}, -6.71 \cdot 10^{-16})$	$(9.38 \cdot 10^{-5}, -1.63 \cdot 10^{-15})$	$(9.84 \cdot 10^{-5}, 7.16 \cdot 10^{-16})$	$(0.000111, -8.82 \cdot 10^{-6})$
	$(1, 2)$	$(3, 4)$	$(4.01, 6)$	$(6, 8)$	$(8.04, 11)$
0.0001	$(5 \cdot 10^{-5}, -3.42 \cdot 10^{-16})$	$(8.75 \cdot 10^{-5}, -9.22 \cdot 10^{-16})$	$(9.38 \cdot 10^{-5}, -1.33 \cdot 10^{-15})$	$(9.84 \cdot 10^{-5}, -4.33 \cdot 10^{-17})$	$(0.000111, -8.82 \cdot 10^{-6})$
	$(1, 2)$	$(3, 5)$	$(4.01, 6)$	$(6, 8)$	$(8.04, 11)$
$1 \cdot 10^{-5}$	$(5 \cdot 10^{-5}, -1.13 \cdot 10^{-16})$	$(8.75 \cdot 10^{-5}, -1.13 \cdot 10^{-15})$	$(9.38 \cdot 10^{-5}, -8.84 \cdot 10^{-16})$	$(9.84 \cdot 10^{-5}, -9.14 \cdot 10^{-16})$	$(0.000111, -8.83 \cdot 10^{-6})$
	$(1, 2)$	$(3, 4)$	$(4, 5)$	$(6, 6)$	$(8.04, 9)$
$1 \cdot 10^{-6}$	$(0.0201, -3.83 \cdot 10^{-5})$	$(8.75 \cdot 10^{-5}, -1.17 \cdot 10^{-15})$	$(9.38 \cdot 10^{-5}, -6.27 \cdot 10^{-16})$	$(9.84 \cdot 10^{-5}, -2.46 \cdot 10^{-15})$	$(0.000111, -8.83 \cdot 10^{-6})$
	$(1, 2)$	$(2, 2)$	$(3.01, 5)$	$(5, 6)$	$(6.91, 9)$

(b) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5 \cdot 10^{-6}, -4.6 \cdot 10^{-17})$	$(8.75 \cdot 10^{-6}, -2 \cdot 10^{-17})$	$(9.38 \cdot 10^{-6}, 3.66 \cdot 10^{-17})$	$(9.84 \cdot 10^{-6}, -2.93 \cdot 10^{-16})$	$(2.5 \cdot 10^{-5}, -9.5 \cdot 10^{-6})$
	$(1, 2)$	$(3.02, 8)$	$(4.03, 13)$	$(6.02, 15)$	$(8.11, 23)$
0.001	$(5 \cdot 10^{-6}, 7.79 \cdot 10^{-17})$	$(8.75 \cdot 10^{-6}, -1.31 \cdot 10^{-16})$	$(9.38 \cdot 10^{-6}, -1.78 \cdot 10^{-17})$	$(9.84 \cdot 10^{-6}, -4.79 \cdot 10^{-17})$	$(2.5 \cdot 10^{-5}, -9.5 \cdot 10^{-6})$
	$(1, 2)$	$(3, 4)$	$(4.01, 6)$	$(6.01, 9)$	$(8.07, 9)$
0.0001	$(5 \cdot 10^{-6}, -4.75 \cdot 10^{-17})$	$(8.75 \cdot 10^{-6}, 5.2 \cdot 10^{-17})$	$(9.38 \cdot 10^{-6}, 4.93 \cdot 10^{-17})$	$(9.84 \cdot 10^{-6}, -2.86 \cdot 10^{-16})$	$(2.5 \cdot 10^{-5}, -9.5 \cdot 10^{-6})$
	$(1, 2)$	$(3, 5)$	$(4.01, 6)$	$(6.01, 9)$	$(8.08, 11)$
$1 \cdot 10^{-5}$	$(5 \cdot 10^{-6}, 3.11 \cdot 10^{-16})$	$(8.75 \cdot 10^{-6}, -6.08 \cdot 10^{-17})$	$(9.38 \cdot 10^{-6}, -8.82 \cdot 10^{-17})$	$(9.84 \cdot 10^{-6}, 3.92 \cdot 10^{-16})$	$(2.5 \cdot 10^{-5}, -9.5 \cdot 10^{-6})$
	$(1, 2)$	$(3, 4)$	$(4, 5)$	$(6, 9)$	$(8.07, 9)$
$1 \cdot 10^{-6}$	$(5 \cdot 10^{-6}, 2.56 \cdot 10^{-17})$	$(8.75 \cdot 10^{-6}, -3.1 \cdot 10^{-17})$	$(9.38 \cdot 10^{-6}, -8.37 \cdot 10^{-17})$	$(9.84 \cdot 10^{-6}, 1.55 \cdot 10^{-16})$	$(2.5 \cdot 10^{-5}, -9.5 \cdot 10^{-6})$
	$(1, 2)$	$(3, 3)$	$(4, 5)$	$(6, 7)$	$(7.91, 9)$

(c) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(5 \cdot 10^{-7}, -8.69 \cdot 10^{-18})$	$(8.75 \cdot 10^{-7}, 2.12 \cdot 10^{-17})$	$(9.37 \cdot 10^{-7}, -7.22 \cdot 10^{-17})$	$(9.84 \cdot 10^{-7}, -2.98 \cdot 10^{-17})$	$(1.62 \cdot 10^{-5}, -9.53 \cdot 10^{-6})$
	$(1, 2)$	$(3.02, 8)$	$(4.02, 10)$	$(6.02, 13)$	$(64.7, 500)$
0.001	$(5 \cdot 10^{-7}, -5.75 \cdot 10^{-17})$	$(8.75 \cdot 10^{-7}, -9.53 \cdot 10^{-17})$	$(9.37 \cdot 10^{-7}, -1.07 \cdot 10^{-17})$	$(9.84 \cdot 10^{-7}, -9.49 \cdot 10^{-18})$	$(1.62 \cdot 10^{-5}, -9.53 \cdot 10^{-6})$
	$(1, 2)$	$(3, 4)$	$(4.01, 6)$	$(6.01, 9)$	$(63.9, 500)$
0.0001	$(5 \cdot 10^{-7}, -4.29 \cdot 10^{-18})$	$(8.75 \cdot 10^{-7}, -5.19 \cdot 10^{-17})$	$(9.37 \cdot 10^{-7}, -1.23 \cdot 10^{-16})$	$(9.84 \cdot 10^{-7}, 2.53 \cdot 10^{-17})$	$(1.62 \cdot 10^{-5}, -9.53 \cdot 10^{-6})$
	$(1, 2)$	$(3, 4)$	$(4.01, 6)$	$(6.01, 9)$	$(58.2, 500)$
$1 \cdot 10^{-5}$	$(5 \cdot 10^{-7}, 1.48 \cdot 10^{-16})$	$(8.75 \cdot 10^{-7}, -5.78 \cdot 10^{-17})$	$(9.37 \cdot 10^{-7}, -6.49 \cdot 10^{-17})$	$(9.84 \cdot 10^{-7}, -9.88 \cdot 10^{-17})$	$(1.62 \cdot 10^{-5}, -9.53 \cdot 10^{-6})$
	$(1, 2)$	$(3, 4)$	$(4, 5)$	$(6, 9)$	$(64.7, 500)$
$1 \cdot 10^{-6}$	$(5 \cdot 10^{-7}, 1.5 \cdot 10^{-16})$	$(8.75 \cdot 10^{-7}, -4.98 \cdot 10^{-18})$	$(9.37 \cdot 10^{-7}, -1.43 \cdot 10^{-17})$	$(9.84 \cdot 10^{-7}, -1.01 \cdot 10^{-16})$	$(1.62 \cdot 10^{-5}, -9.53 \cdot 10^{-6})$
	$(1, 2)$	$(3, 4)$	$(4, 5)$	$(6, 6)$	$(64.5, 500)$

(d) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 4. Cube on a slope, table for $\mu = 0.5$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the average velocity and acceleration ($= 0m \cdot s^{-1}$ in theory), the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(0.000499, -3.23 \cdot 10^{-13})$	$(0.000871, -9.85 \cdot 10^{-13})$	$(0.000928, -1.02 \cdot 10^{-12})$	$(0.00095, -7.1 \cdot 10^{-13})$	$(0.000955, -1.93 \cdot 10^{-13})$
	$(1, 2)$	$(3.02, 14)$	$(4.02, 10)$	$(5.03, 17)$	$(6.03, 18)$
0.001	$(0.000499, -3.23 \cdot 10^{-13})$	$(0.000871, -9.84 \cdot 10^{-13})$	$(0.000928, -1.02 \cdot 10^{-12})$	$(0.00095, -7.12 \cdot 10^{-13})$	$(0.000955, -1.65 \cdot 10^{-13})$
	$(1.01, 3)$	$(3, 5)$	$(4.01, 6)$	$(5.01, 6)$	$(6, 7)$
0.0001	$(0.000499, -3.23 \cdot 10^{-13})$	$(0.000871, -9.86 \cdot 10^{-13})$	$(0.000928, -1.02 \cdot 10^{-12})$	$(0.00095, -7.12 \cdot 10^{-13})$	$(0.000955, -1.74 \cdot 10^{-13})$
	$(1.01, 3)$	$(3, 5)$	$(4.01, 6)$	$(5.01, 7)$	$(6, 7)$
$1 \cdot 10^{-5}$	$(0.000748, -7.33 \cdot 10^{-13})$	$(0.000871, -9.79 \cdot 10^{-13})$	$(0.000928, -1.01 \cdot 10^{-12})$	$(0.00095, -7.04 \cdot 10^{-13})$	$(0.000955, -1.76 \cdot 10^{-13})$
	$(1, 2)$	$(2, 3)$	$(3.01, 5)$	$(4.01, 6)$	$(5.01, 7)$
$1 \cdot 10^{-6}$	$(0.00385, -0.00287)$	$(0.00095, -6.96 \cdot 10^{-13})$	$(0.00095, -6.79 \cdot 10^{-13})$	$(0.00095, -6.65 \cdot 10^{-13})$	$(0.000955, -1.78 \cdot 10^{-13})$
	$(1, 2)$	$(1, 2)$	$(1.01, 4)$	$(1.02, 5)$	$(2.02, 6)$

(a) $\varepsilon_v = 1 \cdot 10^{-3}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(4.99 \cdot 10^{-5}, -3.16 \cdot 10^{-16})$	$(8.7 \cdot 10^{-5}, -8.83 \cdot 10^{-16})$	$(9.27 \cdot 10^{-5}, -1.15 \cdot 10^{-15})$	$(9.55 \cdot 10^{-5}, -1.84 \cdot 10^{-16})$	$(9.55 \cdot 10^{-5}, 4.37 \cdot 10^{-16})$
	$(1, 2)$	$(3.02, 8)$	$(4.02, 11)$	$(6.02, 12)$	$(6.04, 23)$
0.001	$(4.99 \cdot 10^{-5}, -2.05 \cdot 10^{-16})$	$(8.7 \cdot 10^{-5}, -1.07 \cdot 10^{-15})$	$(9.27 \cdot 10^{-5}, -1.33 \cdot 10^{-15})$	$(9.55 \cdot 10^{-5}, 2.09 \cdot 10^{-15})$	$(9.55 \cdot 10^{-5}, 1.84 \cdot 10^{-15})$
	$(1, 2)$	$(3, 3)$	$(4, 5)$	$(6.01, 10)$	$(6.01, 11)$
0.0001	$(4.99 \cdot 10^{-5}, -2.16 \cdot 10^{-16})$	$(8.7 \cdot 10^{-5}, -1.15 \cdot 10^{-15})$	$(9.27 \cdot 10^{-5}, -1.23 \cdot 10^{-15})$	$(9.55 \cdot 10^{-5}, 1.1 \cdot 10^{-15})$	$(9.55 \cdot 10^{-5}, -1.09 \cdot 10^{-15})$
	$(1, 2)$	$(3, 4)$	$(4, 5)$	$(6.01, 8)$	$(6, 8)$
$1 \cdot 10^{-5}$	$(4.99 \cdot 10^{-5}, -1.32 \cdot 10^{-16})$	$(8.7 \cdot 10^{-5}, -8.52 \cdot 10^{-16})$	$(9.27 \cdot 10^{-5}, -1.11 \cdot 10^{-15})$	$(9.55 \cdot 10^{-5}, 1.41 \cdot 10^{-16})$	$(9.55 \cdot 10^{-5}, -2.03 \cdot 10^{-15})$
	$(1, 2)$	$(3, 5)$	$(4, 5)$	$(6, 6)$	$(6, 7)$
$1 \cdot 10^{-6}$	$(0.00311, -0.00299)$	$(8.7 \cdot 10^{-5}, -6.54 \cdot 10^{-16})$	$(9.27 \cdot 10^{-5}, -6.69 \cdot 10^{-16})$	$(9.55 \cdot 10^{-5}, -1.72 \cdot 10^{-15})$	$(9.55 \cdot 10^{-5}, 2.5 \cdot 10^{-15})$
	$(1, 2)$	$(2, 2)$	$(3, 4)$	$(5, 6)$	$(5, 6)$

(b) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(4.99 \cdot 10^{-6}, -2.19 \cdot 10^{-18})$	$(8.7 \cdot 10^{-6}, 4.8 \cdot 10^{-17})$	$(9.27 \cdot 10^{-6}, -1.01 \cdot 10^{-17})$	$(9.55 \cdot 10^{-6}, -2.74 \cdot 10^{-16})$	$(9.55 \cdot 10^{-6}, -2.22 \cdot 10^{-16})$
	$(1, 2)$	$(3.02, 8)$	$(4.03, 13)$	$(6.02, 14)$	$(6.02, 15)$
0.001	$(4.99 \cdot 10^{-6}, 5.53 \cdot 10^{-17})$	$(8.7 \cdot 10^{-6}, 6.81 \cdot 10^{-17})$	$(9.27 \cdot 10^{-6}, 2.22 \cdot 10^{-17})$	$(9.55 \cdot 10^{-6}, -1.13 \cdot 10^{-16})$	$(9.55 \cdot 10^{-6}, 2.06 \cdot 10^{-16})$
	$(1, 2)$	$(3, 4)$	$(4, 5)$	$(6, 8)$	$(6.83, 500)$
0.0001	$(4.99 \cdot 10^{-6}, -5 \cdot 10^{-17})$	$(8.7 \cdot 10^{-6}, 9.23 \cdot 10^{-17})$	$(9.27 \cdot 10^{-6}, -6.77 \cdot 10^{-17})$	$(9.55 \cdot 10^{-6}, -1.03 \cdot 10^{-16})$	$(9.55 \cdot 10^{-6}, -8.5 \cdot 10^{-17})$
	$(1, 2)$	$(3, 4)$	$(4, 5)$	$(6, 8)$	$(6.01, 10)$
$1 \cdot 10^{-5}$	$(4.99 \cdot 10^{-6}, 1.16 \cdot 10^{-16})$	$(8.7 \cdot 10^{-6}, 5.16 \cdot 10^{-17})$	$(9.27 \cdot 10^{-6}, 9.65 \cdot 10^{-17})$	$(9.55 \cdot 10^{-6}, -2.45 \cdot 10^{-17})$	$(9.55 \cdot 10^{-6}, -5.58 \cdot 10^{-17})$
	$(1, 2)$	$(3, 4)$	$(4, 5)$	$(6, 6)$	$(6, 7)$
$1 \cdot 10^{-6}$	$(4.99 \cdot 10^{-6}, -2.12 \cdot 10^{-17})$	$(8.7 \cdot 10^{-6}, -7.53 \cdot 10^{-18})$	$(9.27 \cdot 10^{-6}, 8.13 \cdot 10^{-18})$	$(9.55 \cdot 10^{-6}, -8.72 \cdot 10^{-17})$	$(9.55 \cdot 10^{-6}, -2.25 \cdot 10^{-16})$
	$(1, 2)$	$(3, 3)$	$(4, 5)$	$(5.99, 6)$	$(7.65, 500)$

(c) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(4.99 \cdot 10^{-7}, 1.89 \cdot 10^{-17})$	$(8.7 \cdot 10^{-7}, -4.65 \cdot 10^{-17})$	$(9.27 \cdot 10^{-7}, 2.67 \cdot 10^{-17})$	$(9.55 \cdot 10^{-7}, 6.71 \cdot 10^{-19})$	$(9.55 \cdot 10^{-7}, 4.48 \cdot 10^{-17})$
	$(1, 2)$	$(3.02, 8)$	$(4.03, 15)$	$(6.05, 16)$	$(7.7, 500)$
0.001	$(4.99 \cdot 10^{-7}, -2.5 \cdot 10^{-17})$	$(8.7 \cdot 10^{-7}, -4.61 \cdot 10^{-17})$	$(9.27 \cdot 10^{-7}, 6.92 \cdot 10^{-17})$	$(9.55 \cdot 10^{-7}, -1.22 \cdot 10^{-17})$	$(9.55 \cdot 10^{-7}, 9.3 \cdot 10^{-17})$
	$(1.01, 3)$	$(3, 5)$	$(4, 5)$	$(6, 6)$	$(6.01, 7)$
0.0001	$(4.99 \cdot 10^{-7}, -5.6 \cdot 10^{-17})$	$(8.7 \cdot 10^{-7}, 3.86 \cdot 10^{-17})$	$(9.27 \cdot 10^{-7}, -1.54 \cdot 10^{-17})$	$(9.55 \cdot 10^{-7}, -5.64 \cdot 10^{-18})$	$(9.55 \cdot 10^{-7}, 5.52 \cdot 10^{-17})$
	$(1.01, 3)$	$(3, 5)$	$(4, 5)$	$(6, 7)$	$(7.65, 500)$
$1 \cdot 10^{-5}$	$(4.99 \cdot 10^{-7}, 2.64 \cdot 10^{-17})$	$(8.7 \cdot 10^{-7}, 1.03 \cdot 10^{-17})$	$(9.27 \cdot 10^{-7}, 4.44 \cdot 10^{-18})$	$(9.55 \cdot 10^{-7}, 1.02 \cdot 10^{-16})$	$(9.55 \cdot 10^{-7}, 7.67 \cdot 10^{-17})$
	$(1, 2)$	$(3, 3)$	$(4, 6)$	$(6, 6)$	$(6.01, 7)$
$1 \cdot 10^{-6}$	$(4.99 \cdot 10^{-7}, 4.39 \cdot 10^{-17})$	$(8.7 \cdot 10^{-7}, -4.37 \cdot 10^{-17})$	$(9.27 \cdot 10^{-7}, -3.11 \cdot 10^{-17})$	$(9.55 \cdot 10^{-7}, -8.1 \cdot 10^{-17})$	$(9.55 \cdot 10^{-7}, 5.39 \cdot 10^{-17})$
	$(1.01, 2)$	$(3, 3)$	$(4.01, 6)$	$(6, 7)$	$(6, 7)$

(d) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 5. Cube on a slope, table for $\mu = 0.501$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the average velocity and acceleration (= $0 m \cdot s^{-1}$ in theory), the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(0.000495, -3.15 \cdot 10^{-13})$	$(0.000851, -8.33 \cdot 10^{-13})$	$(0.000892, -5.69 \cdot 10^{-13})$	$(0.0009, -1.32 \cdot 10^{-13})$	$(0.0009, 5.44 \cdot 10^{-15})$
	$(1.01, 4)$	$(3.02, 12)$	$(4.03, 13)$	$(5.05, 14)$	$(6.05, 15)$
0.001	$(0.000495, -3.15 \cdot 10^{-13})$	$(0.000851, -8.37 \cdot 10^{-13})$	$(0.000892, -5.76 \cdot 10^{-13})$	$(0.0009, -1.32 \cdot 10^{-13})$	$(0.0009, -1.23 \cdot 10^{-14})$
	$(1.01, 3)$	$(3, 5)$	$(4, 5)$	$(5.01, 6)$	$(6, 6)$
0.0001	$(0.000495, -3.15 \cdot 10^{-13})$	$(0.000851, -8.31 \cdot 10^{-13})$	$(0.000892, -5.74 \cdot 10^{-13})$	$(0.0009, -1.37 \cdot 10^{-13})$	$(0.0009, 7.16 \cdot 10^{-15})$
	$(1.01, 3)$	$(3, 5)$	$(4, 5)$	$(5.01, 6)$	$(6, 6)$
$1 \cdot 10^{-5}$	$(0.000738, -6.87 \cdot 10^{-13})$	$(0.000851, -8.27 \cdot 10^{-13})$	$(0.000892, -5.76 \cdot 10^{-13})$	$(0.0009, -1.32 \cdot 10^{-13})$	$(0.0009, -8.71 \cdot 10^{-15})$
	$(1, 2)$	$(2.01, 5)$	$(3.01, 6)$	$(4.02, 6)$	$(5.01, 8)$
$1 \cdot 10^{-6}$	$(0.00107, -0.000217)$	$(0.000892, -5.6 \cdot 10^{-13})$	$(0.000892, -5.65 \cdot 10^{-13})$	$(0.0009, -1.24 \cdot 10^{-13})$	$(0.0009, -1.57 \cdot 10^{-14})$
	$(1.01, 2)$	$(1.01, 2)$	$(1.03, 4)$	$(2.02, 4)$	$(3.02, 6)$

(a) $\varepsilon_v = 1 \cdot 10^{-3}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(4.95 \cdot 10^{-5}, -3.36 \cdot 10^{-16})$	$(8.5 \cdot 10^{-5}, -4.67 \cdot 10^{-16})$	$(8.92 \cdot 10^{-5}, -1.35 \cdot 10^{-15})$	$(9 \cdot 10^{-5}, 1.52 \cdot 10^{-17})$	$(9 \cdot 10^{-5}, -4.01 \cdot 10^{-16})$
	$(1.01, 3)$	$(3.02, 8)$	$(4.04, 19)$	$(5.04, 11)$	$(6.07, 21)$
0.001	$(4.95 \cdot 10^{-5}, -2.58 \cdot 10^{-16})$	$(8.5 \cdot 10^{-5}, -4.88 \cdot 10^{-16})$	$(8.92 \cdot 10^{-5}, -1.38 \cdot 10^{-15})$	$(9 \cdot 10^{-5}, -6.38 \cdot 10^{-16})$	$(9 \cdot 10^{-5}, 3.74 \cdot 10^{-16})$
	$(1.01, 3)$	$(3, 4)$	$(4.01, 6)$	$(5.01, 6)$	$(6, 7)$
0.0001	$(4.95 \cdot 10^{-5}, -3.07 \cdot 10^{-16})$	$(8.5 \cdot 10^{-5}, -5.58 \cdot 10^{-16})$	$(8.92 \cdot 10^{-5}, -3.13 \cdot 10^{-16})$	$(9 \cdot 10^{-5}, 3.43 \cdot 10^{-16})$	$(9 \cdot 10^{-5}, -8.22 \cdot 10^{-16})$
	$(1.01, 3)$	$(3, 4)$	$(4.01, 6)$	$(5.01, 6)$	$(6.02, 10)$
$1 \cdot 10^{-5}$	$(4.95 \cdot 10^{-5}, -2.09 \cdot 10^{-16})$	$(8.5 \cdot 10^{-5}, -1.05 \cdot 10^{-15})$	$(8.92 \cdot 10^{-5}, -1.81 \cdot 10^{-15})$	$(9 \cdot 10^{-5}, 1.05 \cdot 10^{-15})$	$(9 \cdot 10^{-5}, -7.82 \cdot 10^{-16})$
	$(1, 3)$	$(3, 5)$	$(4, 6)$	$(5.01, 6)$	$(6, 8)$
$1 \cdot 10^{-6}$	$(0.000216, -0.000176)$	$(8.5 \cdot 10^{-5}, -9.9 \cdot 10^{-16})$	$(8.92 \cdot 10^{-5}, -9.17 \cdot 10^{-16})$	$(9 \cdot 10^{-5}, 3.55 \cdot 10^{-16})$	$(9 \cdot 10^{-5}, -1.39 \cdot 10^{-15})$
	$(1.01, 2)$	$(2.01, 3)$	$(3.01, 4)$	$(4.01, 5)$	$(5.01, 6)$

(b) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(4.95 \cdot 10^{-6}, -2.92 \cdot 10^{-17})$	$(8.5 \cdot 10^{-6}, -1.77 \cdot 10^{-17})$	$(8.92 \cdot 10^{-6}, 1.18 \cdot 10^{-16})$	$(9 \cdot 10^{-6}, 1.18 \cdot 10^{-17})$	$(9 \cdot 10^{-6}, -5.79 \cdot 10^{-11})$
	$(1, 2)$	$(3.02, 9)$	$(4.02, 12)$	$(5.09, 24)$	$(153, 500)$
0.001	$(4.95 \cdot 10^{-6}, -8.72 \cdot 10^{-17})$	$(8.5 \cdot 10^{-6}, 1.95 \cdot 10^{-17})$	$(8.92 \cdot 10^{-6}, 1.21 \cdot 10^{-17})$	$(9 \cdot 10^{-6}, -6.89 \cdot 10^{-17})$	$(9 \cdot 10^{-6}, 1.02 \cdot 10^{-10})$
	$(1.01, 3)$	$(3, 4)$	$(4, 5)$	$(5.01, 7)$	$(153, 500)$
0.0001	$(4.95 \cdot 10^{-6}, -5.06 \cdot 10^{-17})$	$(8.5 \cdot 10^{-6}, -9.55 \cdot 10^{-17})$	$(8.92 \cdot 10^{-6}, -2.55 \cdot 10^{-17})$	$(9 \cdot 10^{-6}, 5.26 \cdot 10^{-19})$	$(9 \cdot 10^{-6}, -5.7 \cdot 10^{-11})$
	$(1.01, 3)$	$(3, 4)$	$(4, 5)$	$(5.01, 7)$	$(146, 500)$
$1 \cdot 10^{-5}$	$(4.95 \cdot 10^{-6}, 1.99 \cdot 10^{-16})$	$(8.5 \cdot 10^{-6}, 4.22 \cdot 10^{-17})$	$(8.92 \cdot 10^{-6}, 5.26 \cdot 10^{-17})$	$(9 \cdot 10^{-6}, 5.43 \cdot 10^{-17})$	$(9 \cdot 10^{-6}, 9.35 \cdot 10^{-11})$
	$(1, 2)$	$(3, 4)$	$(4, 6)$	$(5.01, 6)$	$(161, 500)$
$1 \cdot 10^{-6}$	$(4.95 \cdot 10^{-6}, -1.59 \cdot 10^{-18})$	$(8.5 \cdot 10^{-6}, 4.47 \cdot 10^{-17})$	$(8.92 \cdot 10^{-6}, -1.23 \cdot 10^{-16})$	$(9 \cdot 10^{-6}, -8.76 \cdot 10^{-17})$	$(nan, 0)$
	$(1.01, 2)$	$(3, 3)$	$(4.01, 6)$	$(4.99, 5)$	$(116, 500)$

(c) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	0.1	0.01	0.001	0.0001	$1 \cdot 10^{-5}$
0.01	$(4.95 \cdot 10^{-7}, -1.84 \cdot 10^{-17})$	$(8.5 \cdot 10^{-7}, 4.28 \cdot 10^{-17})$	$(8.92 \cdot 10^{-7}, 7.28 \cdot 10^{-17})$	$(9 \cdot 10^{-7}, -7.49 \cdot 10^{-17})$	$(9 \cdot 10^{-7}, -1.84 \cdot 10^{-11})$
	$(1, 2)$	$(3.02, 11)$	$(4.03, 13)$	$(5.05, 14)$	$(179, 500)$
0.001	$(4.95 \cdot 10^{-7}, 2.45 \cdot 10^{-17})$	$(8.5 \cdot 10^{-7}, -1.05 \cdot 10^{-17})$	$(8.92 \cdot 10^{-7}, -2.73 \cdot 10^{-17})$	$(9 \cdot 10^{-7}, -6.94 \cdot 10^{-17})$	$(9 \cdot 10^{-7}, 1.13 \cdot 10^{-11})$
	$(1.01, 3)$	$(3, 4)$	$(4.01, 6)$	$(5.01, 6)$	$(164, 500)$
0.0001	$(4.95 \cdot 10^{-7}, 3.44 \cdot 10^{-17})$	$(8.5 \cdot 10^{-7}, 2.73 \cdot 10^{-17})$	$(8.92 \cdot 10^{-7}, -1.76 \cdot 10^{-16})$	$(9 \cdot 10^{-7}, -9.97 \cdot 10^{-17})$	$(9 \cdot 10^{-7}, 7.92 \cdot 10^{-12})$
	$(1.01, 3)$	$(3, 4)$	$(4.01, 6)$	$(5.01, 6)$	$(204, 500)$
$1 \cdot 10^{-5}$	$(4.95 \cdot 10^{-7}, 5.25 \cdot 10^{-17})$	$(8.5 \cdot 10^{-7}, 1.92 \cdot 10^{-17})$	$(8.92 \cdot 10^{-7}, 3.7 \cdot 10^{-19})$	$(9 \cdot 10^{-7}, 3.06 \cdot 10^{-17})$	$(9 \cdot 10^{-7}, -4.37 \cdot 10^{-12})$
	$(1, 2)$	$(3, 5)$	$(4, 6)$	$(5, 6)$	$(171, 500)$
$1 \cdot 10^{-6}$	$(4.95 \cdot 10^{-7}, -1.75 \cdot 10^{-17})$	$(8.5 \cdot 10^{-7}, -3.68 \cdot 10^{-17})$	$(8.92 \cdot 10^{-7}, -9.29 \cdot 10^{-17})$	$(9 \cdot 10^{-7}, -6.44 \cdot 10^{-17})$	$(nan, 0)$
	$(1.01, 2)$	$(2.99, 3)$	$(4, 4)$	$(5, 5)$	$(6, 6)$

(d) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 6. Cube on a slope, table for $\mu = 0.505$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the average velocity and acceleration ($= 0m \cdot s^{-1}$ in theory), the second line the average and maximum number of lagged iterations per timestep.

$\mu \backslash \epsilon_\tau$	0.001	0.0001	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
0.45	(1.265833, 0.444760)	(1.038909, 0.396504)	(1.105785, 0.441897)	(1.100753, 0.438557)	(1.101121, 0.438688)	(1.101142, 0.438700)	(1.101142, 0.438699)
0.495	(0.1433403, 0.053750)	(0.107090, 0.041794)	(0.109913, 0.043809)	(0.110058, 0.043833)	(0.110090, 0.043861)	(0.110091, 0.043861)	(0.110092, 0.043861)
0.499	$(6.60 \cdot 10^{-5}, 1.45 \cdot 10^{-6})$	(0.023072, 0.008949)	(0.022052, 0.008757)	(0.022002, 0.008764)	(0.021999, 0.008764)	(0.021998, 0.008764)	(0.021998, 0.008764)
0.5	$(5.52 \cdot 10^{-5}, 1.17 \cdot 10^{-6})$	$(6.68 \cdot 10^{-5}, 3.51 \cdot 10^{-6})$	$(8.48 \cdot 10^{-5}, 1.22 \cdot 10^{-5})$	$(4.11 \cdot 10^{-5}, 3.39 \cdot 10^{-6})$	$(1.15 \cdot 10^{-5}, -1.71 \cdot 10^{-6})$	$(3.34 \cdot 10^{-9}, 4.98 \cdot 10^{-11})$	$(3.39 \cdot 10^{-9}, 4.98 \cdot 10^{-11})$
0.501	$(4.49 \cdot 10^{-5}, 9.14 \cdot 10^{-7})$	$(4.49 \cdot 10^{-5}, 9.14 \cdot 10^{-7})$	$(1.47 \cdot 10^{-5}, 3.29 \cdot 10^{-7})$	$(1.47 \cdot 10^{-5}, 3.29 \cdot 10^{-7})$	$(2.50 \cdot 10^{-6}, 2.03 \cdot 10^{-8})$	$(2.53 \cdot 10^{-7}, 2.68 \cdot 10^{-10})$	$(2.65 \cdot 10^{-9}, 2.65 \cdot 10^{-14})$
0.505	$(4.78 \cdot 10^{-5}, 6.00 \cdot 10^{-8})$	$(1.56 \cdot 10^{-5}, 7.28 \cdot 10^{-8})$	$(1.56 \cdot 10^{-5}, 7.28 \cdot 10^{-8})$	$(1.81 \cdot 10^{-6}, 1.61 \cdot 10^{-9})$	$(1.79 \cdot 10^{-7}, 1.51 \cdot 10^{-11})$	$(1.90 \cdot 10^{-8}, 1.64 \cdot 10^{-13})$	$(2.07 \cdot 10^{-9}, 1.92 \cdot 10^{-15})$

(a) $\epsilon_p = \epsilon_s = 1 \cdot 10^{-8}$

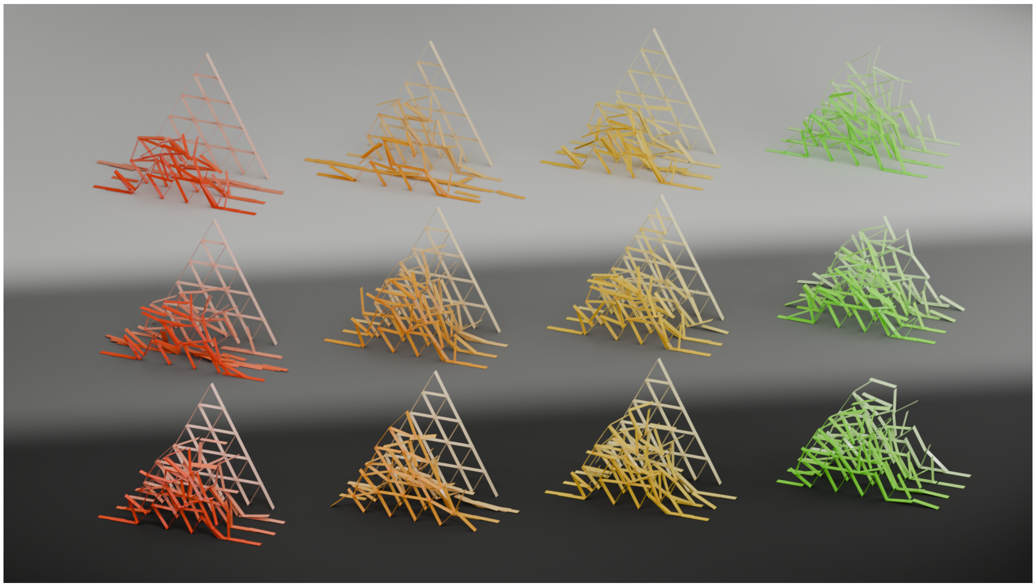
$\mu \backslash \epsilon_\tau$	0.001	0.0001	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
0.45	(1.264473, 0.444320)	(1.038093, 0.395986)	(1.104580, 0.441193)	(1.099543, 0.437832)	(1.099911, 0.437964)	(1.099931, 0.437976)	(1.099931, 0.437975)
0.495	(0.143415, 0.054080)	(0.106972, 0.041728)	(0.109787, 0.043736)	(0.109933, 0.043760)	(0.109964, 0.043787)	(0.109966, 0.043787)	(0.109966, 0.043788)
0.499	$(6.60 \cdot 10^{-5}, 1.45 \cdot 10^{-6})$	(0.023016, 0.008927)	(0.022012, 0.008739)	(0.021959, 0.008744)	(0.021956, 0.008745)	(0.021956, 0.008745)	(0.021956, 0.008745)
0.5	$(5.52 \cdot 10^{-5}, 1.17 \cdot 10^{-6})$	$(6.63 \cdot 10^{-5}, 3.41 \cdot 10^{-6})$	$(7.43 \cdot 10^{-5}, 9.64 \cdot 10^{-6})$	$(4.16 \cdot 10^{-5}, 2.70 \cdot 10^{-6})$	$(1.15 \cdot 10^{-5}, -7.03 \cdot 10^{-9})$	$(8.26 \cdot 10^{-9}, 4.68 \cdot 10^{-11})$	$(1.78 \cdot 10^{-9}, 8.11 \cdot 10^{-12})$
0.501	$(4.49 \cdot 10^{-5}, 9.14 \cdot 10^{-7})$	$(4.49 \cdot 10^{-5}, 9.14 \cdot 10^{-7})$	$(1.47 \cdot 10^{-5}, 3.28 \cdot 10^{-7})$	$(1.47 \cdot 10^{-5}, 3.28 \cdot 10^{-7})$	$(2.49 \cdot 10^{-6}, 2.01 \cdot 10^{-8})$	$(2.52 \cdot 10^{-7}, 2.66 \cdot 10^{-10})$	$(2.64 \cdot 10^{-9}, 2.63 \cdot 10^{-14})$
0.505	$(4.78 \cdot 10^{-5}, 6.00 \cdot 10^{-8})$	$(1.56 \cdot 10^{-5}, 7.27 \cdot 10^{-8})$	$(1.56 \cdot 10^{-5}, 7.27 \cdot 10^{-8})$	$(1.81 \cdot 10^{-6}, 1.61 \cdot 10^{-9})$	$(1.79 \cdot 10^{-7}, 1.50 \cdot 10^{-11})$	$(1.90 \cdot 10^{-8}, 1.63 \cdot 10^{-13})$	$(2.07 \cdot 10^{-9}, 1.92 \cdot 10^{-15})$

(b) $\epsilon_p = \epsilon_s = 1 \cdot 10^{-6}$

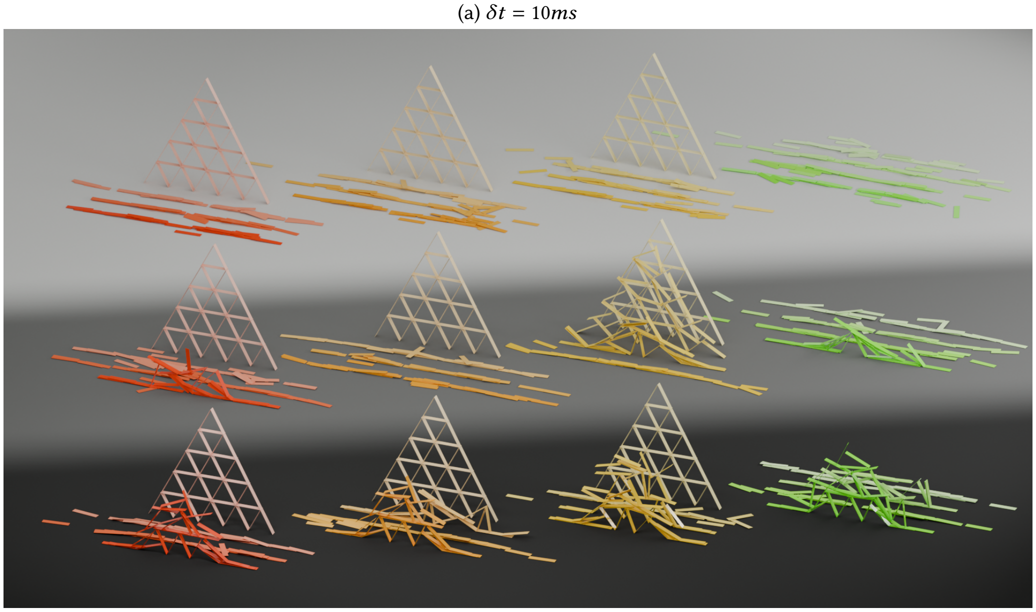
$\mu \backslash \epsilon_\tau$	0.001	0.0001	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
0.45	(1.149519, 0.405515)	(0.941103, 0.344339)	(0.991991, 0.375887)	(0.988164, 0.372930)	(0.988539, 0.373118)	(0.988561, 0.373130)	(0.988561, 0.373130)
0.495	(0.131367, 0.047508)	(0.095962, 0.035548)	(0.098239, 0.037156)	(0.098395, 0.037201)	(0.098417, 0.037220)	(0.098419, 0.037221)	(0.098419, 0.037221)
0.499	$(6.40 \cdot 10^{-5}, 1.40 \cdot 10^{-6})$	(0.019100, 0.007238)	(0.017918, 0.007018)	(0.017842, 0.007017)	(0.017836, 0.007017)	(0.017836, 0.007017)	(0.017835, 0.007017)
0.5	$(5.33 \cdot 10^{-5}, 1.12 \cdot 10^{-6})$	$(5.33 \cdot 10^{-5}, 1.12 \cdot 10^{-6})$	$(3.59 \cdot 10^{-5}, 2.30 \cdot 10^{-6})$	$(3.59 \cdot 10^{-5}, 2.30 \cdot 10^{-6})$	$(1.50 \cdot 10^{-5}, 9.46 \cdot 10^{-7})$	$(7.74 \cdot 10^{-7}, 1.22 \cdot 10^{-8})$	$(8.24 \cdot 10^{-8}, 2.01 \cdot 10^{-10})$
0.501	$(4.31 \cdot 10^{-5}, 8.70 \cdot 10^{-7})$	$(4.31 \cdot 10^{-5}, 8.70 \cdot 10^{-7})$	$(1.27 \cdot 10^{-5}, 2.39 \cdot 10^{-7})$	$(1.27 \cdot 10^{-5}, 2.39 \cdot 10^{-7})$	$(1.91 \cdot 10^{-6}, 1.08 \cdot 10^{-8})$	$(1.85 \cdot 10^{-7}, 1.17 \cdot 10^{-10})$	$(1.86 \cdot 10^{-8}, 1.10 \cdot 10^{-12})$
0.505	$(4.77 \cdot 10^{-5}, 5.97 \cdot 10^{-8})$	$(1.52 \cdot 10^{-5}, 6.73 \cdot 10^{-8})$	$(1.52 \cdot 10^{-5}, 6.73 \cdot 10^{-8})$	$(1.75 \cdot 10^{-6}, 1.42 \cdot 10^{-9})$	$(1.73 \cdot 10^{-7}, 1.33 \cdot 10^{-11})$	$(1.84 \cdot 10^{-8}, 1.45 \cdot 10^{-13})$	$(2.00 \cdot 10^{-9}, 1.71 \cdot 10^{-15})$

(c) $\epsilon_p = \epsilon_s = 1 \cdot 10^{-4}$

Table 7. Cube on a slope, table for $\mu = 0.505$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the average velocity and acceleration (= $0 m \cdot s^{-1}$ in theory), the second line the average and maximum number of lagged iterations per timestep.

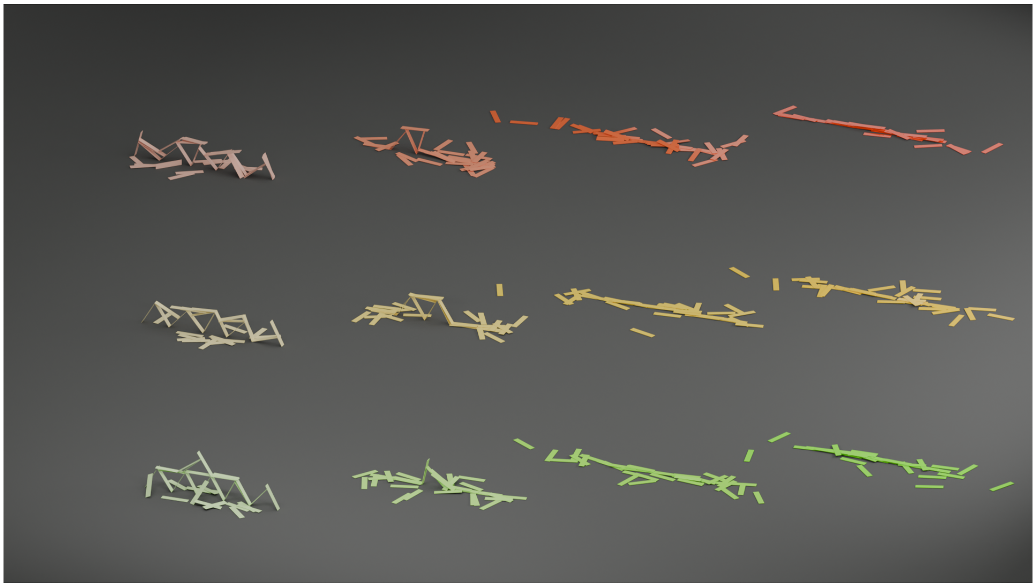


(a) $\delta t = 10ms$

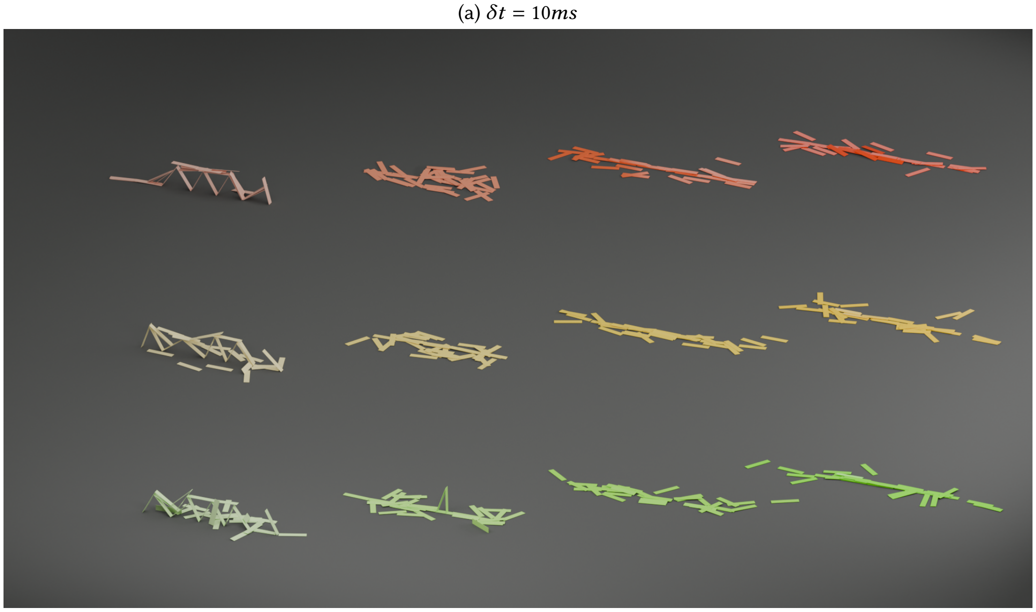


(b) $\delta t = 5ms$

Fig. 3. Houses of cards simulated with Rigid–IPC with $\mu = 0.2$, *last converged frame rendered*. A darker ground denotes a stiffer friction model (smaller ε_v). The saturation (from white to colored) indicates a more accurate Newton solve (lower ε_d tolerance) and the hue (from red to green) a more accurate friction solve (lower ε_μ tolerance). Detailed values are in the tables of Sec. 3.3. A certain level of accuracy is needed to trigger the initial collapse but may prevents convergence in subsequent timesteps.



(a) $\delta t = 10ms$



(b) $\delta t = 5ms$

Fig. 4. Houses of cards simulated with our method with $\mu = 0.2$, on the *last frame*. The saturation (from white to colored) indicates a more accurate Newton solve (lower ϵ_τ tolerance) and the hue (from red to green) a lower perturbation (lower ϵ_p and ϵ_s). Detailed values are in the tables of Sec. 3.3. All examples manage to collapse in these simulations.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	/S	/S	/S	37/TC
	(1, 1)	(1, 1)	(1, 1)	(34.5, 75)
$1 \cdot 10^{-3}$	/TC	39/TC	55/TC	37/TC
	(1, 1)	(1, 1)	(1, 1)	(15.9, 78)
$1 \cdot 10^{-4}$	44/TC	95/TC	59/TC	47/TC
	(1, 1)	(1, 1)	(1, 1)	(6.93, 100)
$1 \cdot 10^{-5}$	40/TC	45/TC	40/TC	37/TC
	(1, 1)	(1, 1)	(1, 1)	(2.61, 6)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	/S	/S	/S	36/TC
	(1, 1)	(1, 1)	(1, 1)	(55.9, 100)
$1 \cdot 10^{-3}$	51/TC	40/TC	29/S	34/TC
	(1, 1)	(1, 1)	(1, 1)	(31.6, 77)
$1 \cdot 10^{-4}$	81/TC	36/TC	43/TC	37/TC
	(1, 1)	(1, 1)	(1, 1)	(6.89, 35)
$1 \cdot 10^{-5}$	39/TC	34/TC	37/TC	40/TC
	(1, 1)	(1, 1)	(1, 1)	(3.23, 9)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	37/TC
	(1, 1)	(1, 1)	(1, 2)	(86.8, 100)
$1 \cdot 10^{-3}$	46/TC	77/TC	51/TC	42/TC
	(1, 1)	(1, 1)	(1, 1)	(65.5, 100)
$1 \cdot 10^{-4}$	48/TC	42/TC	33/TC	36/TC
	(1, 1)	(1, 1)	(1, 1)	(18.9, 40)
$1 \cdot 10^{-5}$	34/TC	20/S	36/TC	36/TC
	(1, 1)	(1, 1)	(1, 1)	(4.49, 14)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 8. House of cards, table for $\delta t = 10ms, \mu = 0.2$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	75/TC
	(1, 1)	(1, 1)	(1, 1)	(41, 100)
$1 \cdot 10^{-3}$	165/CC	167/CC	70/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(7.95, 71)
$1 \cdot 10^{-4}$	92/TC	94/TC	61/TC	64/TC
	(1, 1)	(1, 1)	(1.02, 2)	(6.38, 100)
$1 \cdot 10^{-5}$	60/TC	56/TC	59/TC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(1.97, 7)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	85/TC
	(1, 1)	(1, 1)	(1, 1)	(77.5, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	62/TC
	(1, 1)	(1, 1)	(1, 1)	(36.2, 100)
$1 \cdot 10^{-4}$	58/TC	46/TC	78/TC	114/CC
	(1, 1)	(1, 1)	(1.01, 2)	(12.6, 100)
$1 \cdot 10^{-5}$	61/TC	64/TC	60/TC	-/CC
	(1, 1)	(1, 1)	(1.02, 2)	(4.56, 100)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 1)	(31.6, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	120/CC
	(1, 1)	(1, 1)	(1.01, 3)	(76.1, 100)
$1 \cdot 10^{-4}$	41/TC	47/TC	43/TC	115/CC
	(1, 1)	(1, 1)	(1, 1)	(22.3, 100)
$1 \cdot 10^{-5}$	58/TC	61/TC	58/TC	28/S
	(1, 1)	(1, 1)	(1, 1)	(8.78, 100)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 9. House of cards, table for $\delta t = 10ms, \mu = 0.3$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 2)	(3.8, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	244/PC
	(1, 1)	(1, 1)	(1, 1)	(10.6, 100)
$1 \cdot 10^{-4}$	-/CC	147/CC	149/CC	-/PC
	(1, 1)	(1, 1)	(1.01, 2)	(3.3, 100)
$1 \cdot 10^{-5}$	195/CC	131/CC	-/PC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(2.48, 100)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 2)	(5.51, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 2)	(12.1, 100)
$1 \cdot 10^{-4}$	-/PC	-/PC	-/PC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(5.54, 100)
$1 \cdot 10^{-5}$	155/CC	158/CC	-/PC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(3.09, 100)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(7.47, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1.01, 4)	(27.7, 100)
$1 \cdot 10^{-4}$	-/PC	-/PC	-/PC	3/S
	(1, 1)	(1, 1)	(1, 1)	(25.3, 64)
$1 \cdot 10^{-5}$	-/PC	-/PC	-/PC	3/S
	(1, 1)	(1, 1)	(1, 1)	(12.7, 35)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 10. House of cards, table for $\delta t = 10ms, \mu = 0.4$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 2)	(4.7, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 1)	(7.23, 100)
$1 \cdot 10^{-4}$	-/PC	-/PC	-/PC	-/S
	(1, 1)	(1, 1)	(1, 1)	(4.3, 100)
$1 \cdot 10^{-5}$	-/PC	-/PC	-/PC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(4.38, 100)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(5.57, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 2)	(5.28, 100)
$1 \cdot 10^{-4}$	-/S	-/PC	-/PC	-/S
	(1, 1)	(1, 1)	(1, 1)	(4.98, 100)
$1 \cdot 10^{-5}$	-/PC	-/PC	-/PC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(4.59, 100)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 2)	(6.6, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1.01, 3)	(6.89, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/PC	3/S
	(1, 1)	(1, 1)	(1.01, 3)	(18.7, 50)
$1 \cdot 10^{-5}$	62/S	60/S	-/PC	3/S
	(1, 1)	(1, 1)	(1, 1)	(23, 65)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 11. House of cards, table for $\delta t = 10ms, \mu = 0.5$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(4.59, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(4.16, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1.01, 3)	(4.24, 100)
$1 \cdot 10^{-5}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 2)	(4.19, 100)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(5.26, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(5.38, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(4.94, 100)
$1 \cdot 10^{-5}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1.01, 3)	(5.32, 100)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(6.75, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1.01, 4)	(6.64, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/S	3/S
	(1, 1)	(1, 1)	(1.01, 3)	(8.33, 20)
$1 \cdot 10^{-5}$	-/S	-/S	-/S	3/S
	(1, 1)	(1, 1)	(1.01, 3)	(24.3, 69)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 12. House of cards, table for $\delta t = 10ms, \mu = 0.5$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/TC
	(1, 1)	(1, 1)	(1, 1)	(14.2, 100)
$1 \cdot 10^{-3}$	-/TC	-/TC	-/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(4.63, 38)
$1 \cdot 10^{-4}$	-/TC	-/TC	-/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(1.56, 22)
$1 \cdot 10^{-5}$	-/TC	-/TC	-/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(1.33, 8)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/TC
	(1, 1)	(1, 1)	(1, 1)	(27.1, 100)
$1 \cdot 10^{-3}$	-/TC	-/TC	52/S	-/TC
	(1, 1)	(1, 1)	(1, 1)	(9.49, 100)
$1 \cdot 10^{-4}$	-/TC	-/TC	103/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(2.18, 13)
$1 \cdot 10^{-5}$	125/TC	-/TC	-/TC	109/TC
	(1, 1)	(1, 1)	(1, 1)	(2.19, 8)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/TC
	(1, 1)	(1, 1)	(1, 1)	(38.7, 100)
$1 \cdot 10^{-3}$	-/TC	133/TC	-/TC	106/TC
	(1, 1)	(1, 1)	(1, 1)	(27.6, 100)
$1 \cdot 10^{-4}$	-/TC	-/TC	78/TC	204/TC
	(1, 1)	(1, 1)	(1, 1)	(7.22, 33)
$1 \cdot 10^{-5}$	81/TC	80/TC	91/TC	74/TC
	(1, 1)	(1, 1)	(1, 1)	(1.96, 8)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 13. House of cards, table for $\delta t = 5ms, \mu = 0.2$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/TC
	(1, 1)	(1, 1)	(1, 1)	(17.7, 100)
$1 \cdot 10^{-3}$	-/TC	-/TC	-/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(5.63, 78)
$1 \cdot 10^{-4}$	-/TC	-/TC	-/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(1.94, 47)
$1 \cdot 10^{-5}$	-/TC	-/TC	-/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(1.69, 100)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/TC
	(1, 1)	(1, 1)	(1, 1)	(35.6, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/TC	47/S
	(1, 1)	(1, 1)	(1, 1)	(28.1, 100)
$1 \cdot 10^{-4}$	-/TC	72/TC	-/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(3.4, 43)
$1 \cdot 10^{-5}$	144/TC	-/TC	-/TC	-/TC
	(1, 1)	(1, 1)	(1, 1)	(1.69, 18)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/TC
	(1, 1)	(1, 1)	(1, 1)	(48, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	164/TC
	(1, 1)	(1, 1)	(1, 1)	(53.7, 100)
$1 \cdot 10^{-4}$	111/TC	84/TC	140/TC	182/TC
	(1, 1)	(1, 1)	(1, 1)	(9.25, 100)
$1 \cdot 10^{-5}$	51/S	140/TC	52/S	61/TC
	(1, 1)	(1, 1)	(1, 1)	(2.58, 6)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 14. House of cards, table for $\delta t = 5ms, \mu = 0.3$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 1)	(5.61, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(2.73, 100)
$1 \cdot 10^{-4}$	-/PC	-/PC	-/PC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(2.3, 100)
$1 \cdot 10^{-5}$	-/PC	-/PC	-/PC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(1.92, 100)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(4.71, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 1)	(8.34, 100)
$1 \cdot 10^{-4}$	-/PC	-/PC	-/CC	416/PC
	(1, 1)	(1, 1)	(1, 1)	(4.26, 81)
$1 \cdot 10^{-5}$	-/PC	-/PC	-/PC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(2.31, 100)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 2)	(7.31, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	267/PC
	(1, 1)	(1, 1)	(1, 1)	(45.6, 100)
$1 \cdot 10^{-4}$	66/S	91/S	-/PC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(5.64, 100)
$1 \cdot 10^{-5}$	29/S	48/S	280/CC	-/PC
	(1, 1)	(1, 1)	(1, 1)	(2.39, 100)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 15. House of cards, table for $\delta t = 5ms, \mu = 0.4$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(3.39, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 1)	(5.03, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(2.53, 100)
$1 \cdot 10^{-5}$	-/PC	-/PC	-/PC	-/S
	(1, 1)	(1, 1)	(1, 1)	(2.52, 100)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 2)	(5.04, 100)
$1 \cdot 10^{-3}$	-/S	14/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 1)	(10.3, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(3.09, 100)
$1 \cdot 10^{-5}$	-/PC	-/PC	-/PC	-/S
	(1, 1)	(1, 1)	(1, 1)	(2.57, 100)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(9.06, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 1)	(20.8, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(4.27, 100)
$1 \cdot 10^{-5}$	140/S	-/PC	-/PC	-/S
	(1, 1)	(1, 1)	(1, 1)	(2.65, 100)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 16. House of cards, table for $\delta t = 5ms, \mu = 0.5$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(3.46, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(3.24, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(3.23, 100)
$1 \cdot 10^{-5}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(3.33, 100)

(a) $\varepsilon_v = 1 \cdot 10^{-4}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(4.88, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 1)	(10.9, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(3.82, 100)
$1 \cdot 10^{-5}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(3.35, 100)

(b) $\varepsilon_v = 1 \cdot 10^{-5}$

$\varepsilon_d \backslash \varepsilon_\mu$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-8}$
$1 \cdot 10^{-2}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(6.21, 100)
$1 \cdot 10^{-3}$	-/S	-/S	-/S	9/S
	(1, 1)	(1, 1)	(1, 1)	(39.8, 100)
$1 \cdot 10^{-4}$	-/S	-/S	-/S	-/S
	(1, 1)	(1, 1)	(1, 1)	(4.5, 100)
$1 \cdot 10^{-5}$	-/S	-/S	-/S	-/PC
	(1, 1)	(1, 1)	(1, 1)	(4.03, 100)

(c) $\varepsilon_v = 1 \cdot 10^{-6}$

Table 17. House of cards, table for $\delta t = 5ms, \mu = 0.5$. For a couple $(\varepsilon_d, \varepsilon_\mu)$, the first line gives the last non-converged timestep if the simulation ended early and the observed behavior (TC, CC, PC or S) and the second line the average and maximum number of lagged iterations per timestep.

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	TC	TC	TC	TC
10^{-6}	TC	TC	TC	TC
10^{-8}	TC	TC	TC	TC

(a) $\mu = 0.2$

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	TC	TC	TC	TC
10^{-6}	TC	TC	TC	TC
10^{-8}	TC	TC	TC	TC

(b) $\mu = 0.3$

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	TC	CC	CC	CC
10^{-6}	TC	PC	TC	TC
10^{-8}	TC	CC	TC	CC

(c) $\mu = 0.4$

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	S	PC	PC	PC
10^{-6}	S	PC	PC	PC
10^{-8}	PC	PC	PC	PC

(d) $\mu = 0.5$

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	PC	S	S	S
10^{-6}	TC	S	S	S
10^{-8}	TC	S	S	S

(e) $\mu = 0.6$

Table 18. House of cards, table for $\delta t = 10ms$. For a couple μ and ϵ_p, ϵ_d, with the observed behavior (TC, CC, PC or S).

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	TC	TC	TC	TC
10^{-6}	TC	TC	TC	TC
10^{-8}	TC	TC	TC	TC

(a) $\mu = 0.2$

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	TC	TC	TC	TC
10^{-6}	TC	TC	TC	TC
10^{-8}	TC	TC	TC	TC

(b) $\mu = 0.3$

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	TC	CC	CC	CC
10^{-6}	TC	CC	CC	CC
10^{-8}	TC	CC	CC	CC

(c) $\mu = 0.4$

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	TC	PC	PC	PC
10^{-6}	TC	PC	PC	PC
10^{-8}	TC	PC	PC	PC

(d) $\mu = 0.5$

ϵ_p,ϵ_s \tol	1e-2	1e-3	1e-4	1e-5
10^{-4}	TC	PC	S	S
10^{-6}	TC	PC	S	S
10^{-8}	TC	PC	S	S

(e) $\mu = 0.6$

Table 19. House of cards, table for $\delta t = 5ms$. For a couple μ and ϵ_p, ϵ_d, with the observed behavior (TC, CC, PC or S).