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Corrigendum

Volume **38**, Number 1 (1989), in the article "Topologically Sweeping an Arrangement," by Herbert Edelsbrunner and Leonidas J. Guibas, pages 165–194: The proof of Theorem 3.1 given in the paper is incorrect. The amortization argument given there is based on estimating changes in the path length of the upper horizon tree during an elementary step. The difficulty lies in estimating the change from s_i^+ to σ_{i+1}^+, in the notation of that paper. As Fig. 3.4 shows, the change in path length for that leaf can in fact be highly negative. These possible negative changes are not accounted for by the argument given in the paper.

We remark that the proof given in the paper remains valid if we do the bay traversal in the opposite direction (e.g., clockwise for the upper horizon tree). However, this complicates the implementation of the algorithm. Fortunately, a slighty longer proof that was in an earlier version of the paper [*EG*2] is valid for the original traversals and is presented below. Thus Theorem 3.1 is in fact correct as stated and the algorithm and all subsequent results in the paper remain unaffected by the above error in the proof.

The New Proof

Consider an arrangement $\mathscr{A}(H)$ of size n and let l be a line in H. We will obtain a linear bound for the total cost of all bay traversals required during updates to the upper horizon three which involve elementary steps at points of l. The argument that follows is similar to that used in [CGL or EOS] to prove that in an arrangement the total size of all regions bordering a specific line, such as l, is linear.

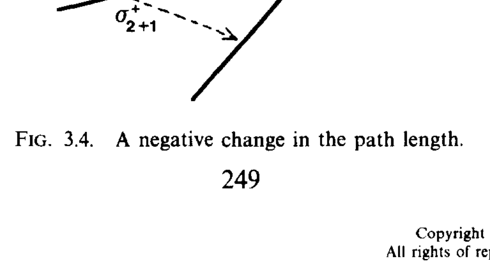

FIG. 3.4. A negative change in the path length.

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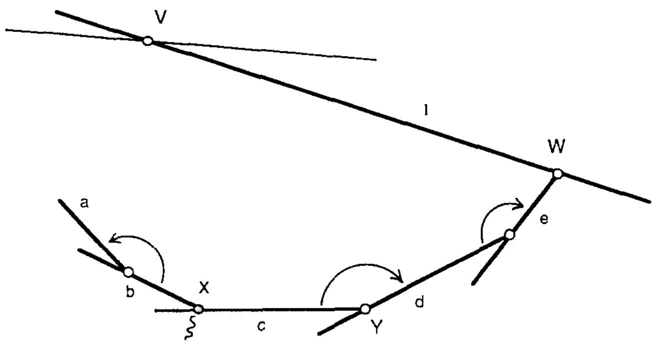

FIG. 3.5. An accounting scheme for bay traversal.

LEMMA 3.1. *In an arrangement $\mathscr{A}(H)$ of n lines, the total number of all edges traversed in the upper horizon tree while performing elementary steps at the vertices lying on line l in H is $O(n)$.*[1]

Proof. Consider the particular bay depicted in Fig. 3.5 associated with the elementary step at vertex V on the line l. We traverse the sequence of edges a, b, etc., proceeding in a counterclockwise fashion, until we come to the edge e intersected by l. How are we to account for the cost of traversing these edges? The first and last edges of such a traversal, such as a and e in our example, are easy to deal with. There are exactly $n-1$ elementary steps performed on vertices of l, so the number of bays traversed in total is only $n-1$ as well. The first and last vertex of each bay traversal can therefore be charged to the corresponding elementary step vertex.

Each bay is convex sequence of edges of monotonically increasing slope, so it contains at most one vertex where there is a supporting line parallel to l. In our example of Fig. 3.5 this is vertex X: all edges of the bay before X have smaller slope than l, and all edges after X have higher slope than l. We will charge the traversal of an edge m to the line containing the *previous* edge if the slope of m is *less* than that of l, else we will charge it to the line containing *next* edge. This leaves the last, and possibly the first, edges of the bay without anyone to charge, but these edges have already been accounted for separately. In our example, b charges a, while c charges d, and d charges e.

We now claim that a particular line can be charged at most once in all the bay traversals associated with l. We deal first with the case of a line such d, whose slope is greater than l. Figure 3.5 depicts a situation where line d is charged by a preceding edge c. Suppose that this is the last time line d will be charged during the traversals under consideration. Note also that, as a consequence of our charging scheme, any line charging d must have slope between those of lines l and d.

[1] The implied constant is $\leqslant 10$. See the references mentioned above.

By the remark at the end of Section 2, each bay of interest in the upper horizon tree bounds a region formed by the intersection of the half-plane below l with all the halfplanes above the lines following l in the current cut. At the current elementary step the intersection Y of c and d, and the intersection of c with l have not been traversed yet. Therefore, in all earlier cuts during the topological sweep, lines l, c, and d will occur in the cut in that order. This implies that, during the bay traversals associated with these earlier cuts, the portion of the line d to the left of Y is shielded by c and therefore cannot be part of any bay and receive charges. The portion of d to the right of Y cannot be charged either. For this to happen, there must be a line c' intersecting d to the right of Y, as part of an earlier bay. But, by our slope condition, c' must intersect l below the current bay's intersection with l at W. It follows that c' occurs below l in the current cut and therefore it is impossible for Y to be part of the current bay, as it is shielded by c', a contradiction.

An entirely analogous argument holds for lines, such as a, of slope less than l. Only now we look at the first time such a line is charged during the bay traversals asociated with propagating l, and then argue that at no later time can our line be charged again. This completes our proof of the linearity. of the total size of the bay traversals performed during all elementary steps lying on a given line l. ∎

This lemma implies that the total cost of the updating the upper horizon tree during the topological sweep is $O(n^2)$. An analogous argument holds for the lower horizon tree. Thus we have shown that we can push our cut from the left all the way to the right in $O(n^2)$ time. At any one time the storage used by our algorithm is $O(n)$, as all our data structures are of linear size.

THEOREM 3.1. *The total cost of updating HTU (or HTL) through all the elementary steps is $O(n^2)$. Therefoe the topological sweep can be carried out in $O(n^2)$ time and $O(n)$ extra storage.*

REFERENCES

[CGL] B. M. CHAZELLE, L. J. GUIBAS, AND D. T. LEE, The power of geometric duality, *BIT* **25** (1985), 76–90.

[EG1] H. EDELSBRUNNER AND L. GUIBAS, Topologically sweeping an arrangement, *J. Comput. System Sci.* **38**, No. 1 (1989), 165–194.

[EG2] H. EDELSBRUNNER AND L. GUIBAS, "Topologically Sweeping an Arrangement," Research Report 9, Digital Systems Research Center, 1986.

[EOS] H. EDELSBRUNNER, J. O'ROURKE, AND R. SEIDEL, Constructing arrangements of lines and hyperplanes with applications, *SIAM J. Comput.* **15** (1986), 317–340.

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