Estimating Adaptive Autoregressive Parameters of Biosignals

A. Schloegl
Department of Medical Informatics, Institute for Biomedical Engineering
University of Technology Graz, Inffeldgasse 16a, A-8010 Graz, Austria
a.schloegl@ieee.org

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Introduction - The AAR model

Different algorithms (LMS, RLS, Recursive AR techniques RAR, and Kalman Filtering KF[1-7]) for the adaptive estimation of Autoregressive parameters are compared.

$$y(t) = a_1(t)*y(t-1) + ... + a_p(t)*y(t-p) + v(t) =$$
$$= \mathbf{a}(t)^T*\mathbf{Y}(t-1) + v(t)$$

The Autoregressive parameters vary with time. An update coefficient *UC* determines the upper limit of the speed of change.

The criterion: one-step prediction error

$$e(t) = y(t) - \hat{a}(t-1)T * Y(t-1)$$

The prediction error e(t) is uncorrelated to all previous samples y(t-i), i>0. The smaller the mean squared error MSE= E{e(t)²}, the better are the estimates â(t).

The estimation methods

LMS I [1, 6]:

$\hat{\mathbf{a}}(t) = \hat{\mathbf{a}}(t-1) + UC/MSY*e(t)*\mathbf{Y}(t-1)$

LMS II ([3], modified):

$\hat{\mathbf{a}}(t) = \hat{\mathbf{a}}(t-1) + UC/V(t)*e(t)*\mathbf{Y}(t-1)$

RAR I ([1, 2], exponential forgetting function):

$\mathbf{A}(t) = (1-UC)*\mathbf{A}(t)+UC*\mathbf{Y}(t)*\mathbf{Y}(t)^T$
$\mathbf{k}(t) = UC*\mathbf{A}(t)*\mathbf{Y}(t)/(UC*\mathbf{Y}(t)^T*\mathbf{A}(t)*\mathbf{Y}(t)+1)$
$\hat{\mathbf{a}}(t) = \hat{\mathbf{a}}(t-1) + \mathbf{k}(t)^T*e(t)$

RAR II ([2], whale forgetting function):

$\mathbf{A}(t) = c1*\mathbf{A}(t-1)+c2*\mathbf{A}(t-2)+c3*\mathbf{Y}(t)*\mathbf{Y}(t)^T$
$(1-(1-2*UC)*z\hat{}-1)^2 = 1 + c1*(z\hat{}-1) + c2*(z\hat{}-2)$
$c3 = 1+c1+c2$

Kalman Filtering (KF) [5, 6, 7]:

$\mathbf{a}(t) = \mathbf{a}(t-1) + \mathbf{w}(t)$
$\mathbf{w}(t) = N(0,\mathbf{W}(t))$

(KF1): RLS; (KF2): [4], (KF3) $\mathbf{W}(t)=UC*trace(\mathbf{A}(t-1))/p$, (KF4): $\mathbf{W}(t)=UC*\mathbf{I}$, (KF5): $\mathbf{W}(t)=UC^2*\mathbf{I}$,

$\mathbf{Q}(t) = \mathbf{Y}(t-1)^T * \mathbf{A}(t-1)*\mathbf{Y}(t-1) + V(t)$
$\mathbf{k}(t) = \mathbf{A}(t-1) * \mathbf{Y}(t-1) / \mathbf{Q}(t)$
$\hat{\mathbf{a}}(t) = \hat{\mathbf{a}}(t-1) + \mathbf{k}(t)^T * e(t)$
$\mathbf{A}(t) = \mathbf{A}(t-1) - \mathbf{k}(t)*\mathbf{Y}(t-1)^T*\mathbf{A}(t-1) + \mathbf{W}(t)$

RLS = KF1 (special form of KF) [4, 6, 7]

$V(t) = 1/(1-UC)$
$\mathbf{W}(t) = UC*\mathbf{A}(t-1)$

Results

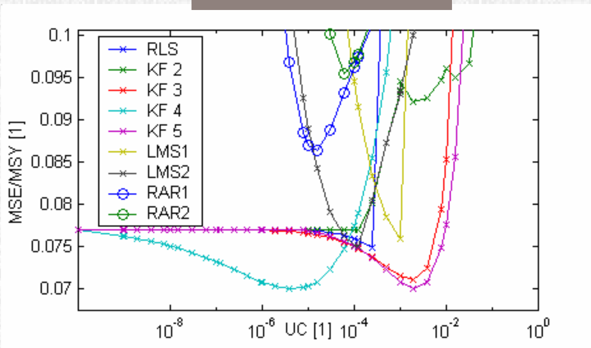

Figure 1: Comparison of AAR estimation algorithms. All algorithms were applied to a non-stationary EEG of 1000s length and sampled with 100Hz. A model order of p=10 was used. The update coefficient *UC* was varied in a range over 10 decades. The lowest error rate is reached with Kalman filtering (KF2 and KF3 with *UC*=2e-3, KF4 and *UC*=4e-6). (See also [6])

Discussion & Conclusion

Forty different UC's were investigated. One optimal *UC*, yielding the lowest MSE, could be identified in each method.

The best method for estimating AAR parameters is Kalman Filtering.

References

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